Homework 9

Name: **SOLUTIONS** Date: June 16, 2015

P 1. Find the point on the graph of $f(x) = (x-1)^2$ that is closest to the point $(-5, 3)$.

Solution: The primary equation is

$$D^2 = (x - x_2)^2 + (y - y_2)^2$$

The secondary equations are

$$\begin{aligned} x_2 &= -5 \\ y_2 &= 3 \\ y &= (x-1)^2 \end{aligned}$$

We use the secondary equations to reduce the primary equation to an equation involving D and x. So our primary equation becomes

$$D^2 = (x+5)^2 + ((x-1)^2 - 3)^2, \quad \text{D: } (-\infty, \infty)$$

Now we find the extrema.

(a) $\frac{dD}{dx} = 0$ at $x = -1$:

$$\begin{aligned} 2D\frac{dD}{dx} &= 2(x+5) + 2((x-1)^2 - 3)2(x-1) \\ \frac{dD}{dx} &= \frac{4x^3 - 12x^2 + 2x + 18}{2D} \\ \frac{dD}{dx} &= \frac{2x^3 - 6x^2 + x + 9}{D} \\ \frac{dD}{dx} &= \frac{(x+1)\left(2x^2 - 8x + 9\right)}{D} \\ (x+1)&\left(2x^2 - 8x + 9\right) = 0 \Rightarrow x = -1 \end{aligned}$$

(b) $\frac{dD}{dx}$ exists for all x.

Now, $f'(x) < 0$ for $x < -1$ and $f'(x) > 0$ for $x > -1$. So f is decreasing to the left of -1 and increasing to the right of -1. Therefore at $x = -1$, f has a global minimum of $D(-1) = \sqrt{85}$.

So the point that is closest to $(-5, 3)$ and lies on the graph of $f(x) = (x-1)^2$ is $(-1, 4)$.

P 2. Approximate $\sqrt[3]{26}$ using the linear approximation of $f(x) = \sqrt[3]{x}$ at $x = 3$.

Solution:

$$f'(x) = \frac{1}{3x^{2/3}}$$

$$f'(3) = \frac{1}{3^{5/3}}$$

$$f(3) = \sqrt[3]{3}$$

So the linear approximation of $f(x)$ at $x = 3$ is

$$L(x) = f(3) + f'(3)(x-3) = \sqrt[3]{3} + \frac{1}{3^{5/3}}(x-3).$$

Note, $f(26) = \sqrt[3]{26} \approx 2.9625$ and $L(26) = 5.128$, which is a bad approximation of $f(26)$. This is to be expected, as $x = 3$ is sufficiently far from $x = 26$, the place where we are trying to approximate the function.