Homework 9

Name: **SOLUTIONS** Date: August 4, 2015

P 1. Determine whether the series converges absolutely or conditional, or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)\cos(\pi n)}$$

Identify the test(s) used and show the conditions of the test hold.

Solution: Note, since $\cos(\pi n) = (-1)^n$

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)\cos(\pi n)} = \sum_{n=1}^{\infty} \frac{1}{(n+1)(-1)^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}.$$

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$ is not absolutely convergent.

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n+1} \right| = \sum_{n=1}^{\infty} \frac{1}{n+1}$$

which diverges by the Limit Comparison Test, with comparison series $\sum_{n=1}^{\infty} \frac{1}{n}$.

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$ is convergent by the Alternating Series Test.

(i) $a_n = 1/(n+1) > 0$.

(ii) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n+1} = 0$.

(iii) $a_{n+1} \le a_n$ for all $n \ge 1$:

$$a_{n+1} \le a_n \Leftrightarrow \frac{1}{n+2} \le \frac{1}{n+1} \Rightarrow n+1 \le n+2 \Rightarrow 0 \le 1$$

So, $\sum_{n=1}^{\infty} \frac{1}{(n+1)\cos(\pi n)}$ is conditionally convergent.

P 2. Determine whether the series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{n3^{n+1}}{(2n)!}$$

Identify the test(s) used and show the conditions of the test hold.

Solution: $\sum_{n=1}^{\infty} \frac{n3^{n+1}}{(2n)!}$ converges absolutely by the Ratio Test.

(a) $a_n = \frac{n3^{n+1}}{(2n)!} \neq 0$ for $n \geq 1$

(b)

$$\begin{aligned}
\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty} \left|\frac{\frac{(n+1)3^{(n+1)+1}}{(2(n+1))!}}{\frac{n3^{n+1}}{(2n)!}}\right| \\
&= \lim_{n\to\infty} \left|\frac{(n+1)3^{n+2}}{(2n+2)!} \cdot \frac{(2n)!}{n3^{n+1}}\right| \\
&= \lim_{n\to\infty} \left|\frac{(n+1)3^{n+1} \cdot 3}{(2n+2)(2n+1)(2n)!} \cdot \frac{(2n)!}{n3^{n+1}}\right| \\
&= \lim_{n\to\infty} \left|\frac{(n+1) \cdot 3}{(2n+2)(2n+1)} \cdot \frac{1}{n}\right| \\
&= \lim_{n\to\infty} \frac{(n+1) \cdot 3}{2(n+1)(2n+1)} \cdot \frac{1}{n} \\
&= \lim_{n\to\infty} \frac{3}{2(2n+1)n} = 0 < 1
\end{aligned}$$

So,

$$\sum_{n=1}^{\infty} \frac{n3^{n+1}}{(2n)!}$$

converges absolutely and so converges.