Homework 8

Name: **SOLUTIONS** Date: June 18, 2015

P 1. Sketch a graph of

$$f(x) = -2 + e^{3x}(4 - 2x)$$

Label any intercepts, relative extrema, points of inflection, and asymptotes.

Solution:

(a) Domain is $(-\infty, \infty)$.

(b) There are no vertical asymptotes.

(c) There is a horizontal asymptote of $y = -2$ to the left.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} (-2 + e^{3x}(4 - 2x)) = -\infty$$
$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} (-2 + e^{3x}(4 - 2x)) = -2$$

(d) $f(x)$ is decreasing on the interval $(5/3, \infty)$.

$$f'(x) = -2e^{3x}(3x - 5)$$

So, $f'(x) = 0$ at $x = 5/3$, $f'(x) > 0$ for $x < 5/3$ and $f'(x) < 0$ for $x > 5/3$.

(e) $f(x)$ is increasing on the interval $(-\infty, 5/3)$.

(f) $f(x)$ is concave up on the interval $(-\infty, 4/3)$.

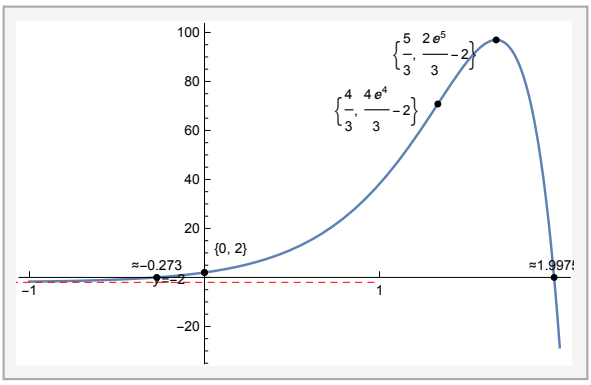
$$f''(x) = -6e^{3x}(3x - 4)$$

So, $f''(x) = 0$ at $x = 4/3$, $f''(x) > 0$ for $x < 4/3$ and $f''(x) < 0$ for $x > 4/3$.

(g) $f(x)$ is concave down on the interval $(4/3, \infty)$.

(h) Inflection Point: $(4/3, 4e^4/3 - 2)$

(i) global maximum of $f(5/3) = 2e^5/3 - 2$.



P 2. Sketch a graph of

$$y = \frac{x}{2} + \ln\left(\frac{x}{x+3}\right)$$

Label any intercepts, relative extrema, points of inflection, and asymptotes.

Solution:

(a) Domain is $(-\infty, -3) \cup (0, \infty)$.

(b) There are vertical asymptotes at $x = 0$ and $x = -3$.

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \left(\frac{x}{2} + \ln\left(\frac{x}{x+3}\right)\right) = -\infty$$
$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \left(\frac{x}{2} + \ln\left(\frac{x}{x+3}\right)\right) = \infty$$

(c) There is are no horizontal asymptotes.

(d) $f(x)$ is always increasing on the intervals $(-\infty, -3) \cup (0, \infty)$.

$$f'(x) = \frac{x^2 + 3x + 6}{2x(x+3)}$$

So, $f'(x) > 0$ for any x in $(-\infty, -3) \cup (0, \infty)$.

(e) $f(x)$ is concave down on the intervals $(-\infty, -3) \cup (0, \infty)$.

$$f''(x) = -\frac{3(2x+3)}{x^2(x+3)^2}$$

So, $f''(x) < 0$ for any x in $(-\infty, -3) \cup (0, \infty)$.

(f) There are no inflection points.

(g) There are no relative extrema.