# Homework 8-2

Name: **SOLUTIONS** Date: August 4, 2015

**P 1.** Determine whether the series converges or diverges, explain.

$$\sum_{n=1}^{\infty} \frac{5n^2+2n-1}{(3n+2)(4n^2+1)n}$$

***Solution***:
Limit Comparison Test:

- Let $a_n = \frac{5n^2+2n-1}{(3n+2)(4n^2+1)n}$.
- Comparison Series: The series

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{5n^2}{12n^4} = \sum_{n=1}^{\infty} \frac{5}{12n^2}$$

  converges by the $p$-series test $(p = 2)$.
- $0 < a_n, b_n$
- $$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{5n^2+2n-1}{(3n+2)(4n^2+1)n} \cdot \frac{12n^2}{5} = 1$$

So, by the Limit Comparison Test,

$$\sum_{n=1}^{\infty} \frac{5n^2+2n-1}{(3n+2)(4n^2+1)n}$$

converges.

**P 2.** Determine whether the series converges or diverges, explain.

$$\sum_{n=1}^{\infty} \pi^{-n^2}$$

***Solution***:
Direct Comparison Test:

- Let $a_n = \pi^{-n^2}$.
- Comparison Series: The series
$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{\pi^n}$$
converges by the Geometric Series Test ($|r| = 1/\pi < 1$).
- $0 < a_n \le b_n$:
$$\pi^{-n^2} \le \frac{1}{\pi^n} \Leftrightarrow \pi^n \le \pi^{n^2} \Leftrightarrow \sqrt[n]{\pi^n} \le \sqrt[n]{\pi^{n^2}} \Leftrightarrow \pi \le \pi^n$$

So, by the Direct Comparison Test,
$$\sum_{n=1}^{\infty} \pi^{-n^2}$$
converges.