Homework 7

Name: **SOLUTIONS** Date: June 10, 2015

P 1. Find the intervals on which

$$f(x) = \frac{\sin x}{1 + \cos^2 x}$$

on the interval $(-2\pi, 2\pi)$ is increasing or decreasing.

Solution:

$$f'(x) = \frac{\cos(x)\left(2\sin^2(x) + \cos^2(x) + 1\right)}{\left(\cos^2(x) + 1\right)^2}$$

If $f'(x) = 0$ then the numerator must be zero.

$$\cos(x)\left(2\sin^2(x) + \cos^2(x) + 1\right) = 0 \Leftrightarrow \cos(x) = 0$$

since $2\sin^2(x) + \cos^2(x) + 1$ is always positive.

Cosine is zero at $x = \pm\pi/2, \pm 3\pi/2$ on the interval $(-2\pi, 2\pi)$.

The domain of $f(x)$ is $(-\infty, \infty)$. So we test the derivative on the intervals $(-2\pi, -3\pi/2)$, $(-3\pi/2, -\pi/2)$, $(-\pi/2, \pi/2)$, $(\pi/2, 3\pi/2)$, and $(3\pi/2, 2\pi)$.

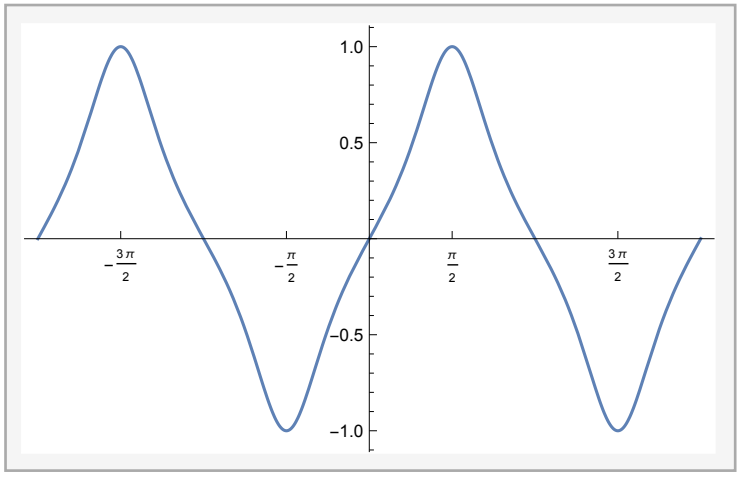
We test the derivative at $x = -7\pi/4, -\pi, 0, \pi, 7\pi/4$. The sign of the derivative at these x-values is positive, negative, positive, negative, positive, respectively.

So, $f(x)$ is increasing on

$$(-2\pi, -3\pi/2) \cup (-\pi/2, \pi/2) \cup (3\pi/2, 2\pi)$$

and $f(x)$ is decreasing on

$$(-3\pi/2, -\pi/2) \cup (\pi/2, 3\pi/2).$$



P 2. Find the points of inflection and discuss the concavity of the graph of

$$y = x - \ln x$$

Solution:

$$f''(x) = \frac{1}{x^2}$$

Since $f''(x) \neq 0$ there are no inflection points.