Homework 7

Name: **SOLUTIONS** Date: July 28, 2015

P 1. Consider the series

$$\sum_{n=1}^{\infty} \frac{2\left(n^2+3n+3\right)}{n(n+1)(n+2)(n+3)}$$

1. Find a formula for s_n. [Hint: Find the partial fractions decomposition of $\frac{2\left(n^2+3n+3\right)}{n(n+1)(n+2)(n+3)}$]
2. Find s_1, s_2, s_3, and s_4
3. Find the sum of the series, S.

Solution: Note,

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} + \frac{1}{n+2} - \frac{1}{n+3}\right)$$

1.
$$\begin{aligned}
s_1 &= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \\
s_2 &= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} = 1 - \frac{1}{5} \\
s_3 &= 1 - \frac{1}{5} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} = 1 + \frac{1}{3} - \frac{1}{4} - \frac{1}{6} \\
s_4 &= 1 + \frac{1}{3} - \frac{1}{4} - \frac{1}{6} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} = 1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7}
\end{aligned}$$

2. $s_n = 1 + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+3}$ for $n \geq 1$

3. $S = \lim\limits_{n\to\infty} s_n = \lim\limits_{n\to\infty} \left(1 + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+3}\right) = 1 + 1/3 = 4/3$

P 2. Consider the sequence with nth term

$$a_n = f^{(n)}(x)$$

where $f(x) = \frac{1}{1-2x}$.

(a) Find the first five terms of the sequence.

(b) Find a formula for the nth term of the sequence.

(c) Simplify $a_n/n!$

Solution:

(a)

$$\begin{aligned}
n = 1 &\Rightarrow f'(x) = \frac{2}{(1-2x)^2} = \frac{2^1 \cdot 1!}{(1-2x)^2} \\
n = 2 &\Rightarrow f''(x) = \frac{8}{(1-2x)^3} = \frac{2^2 \cdot 2!}{(1-2x)^3} \\
n = 3 &\Rightarrow f'''(x) = \frac{48}{(1-2x)^4} = \frac{2^3 \cdot 3!}{(1-2x)^4} \\
n = 4 &\Rightarrow f^{(4)}(x) = \frac{384}{(1-2x)^5} = \frac{2^4 \cdot 4!}{(1-2x)^5} \\
n = 5 &\Rightarrow f^{(5)}(x) = \frac{3840}{(1-2x)^6} = \frac{2^5 \cdot 5!}{(1-2x)^6}
\end{aligned}$$

(b) $f^{(n)}(x) = \dfrac{2^n \cdot n!}{(1-2x)^{n+1}}$

(c) $\dfrac{a_n}{n!} = -\dfrac{\frac{2^n \cdot n!}{(1-2x)^{n+1}}}{n!} = \dfrac{2^n}{(1-2x)^{n+1}}$