Homework 4

Name: **SOLUTIONS** Date: June 2, 2015

P 1. Find an equation for the tangent line to the graph of

$$y = 4 - x^2 - \ln\left(\frac{1}{2}x + 1\right)$$

at $x = 0$.

Solution:

$$\begin{aligned}
y' &= \frac{d}{dx}\left[4 - x^2 - \ln\left(\frac{1}{2}x + 1\right)\right] \\
&= -2x - \frac{d}{dx}\left[\ln\left(\frac{1}{2}x + 1\right)\right] \\
&= -2x - \frac{1}{\frac{1}{2}x + 1}\frac{d}{dx}\left[\frac{1}{2}x + 1\right] \\
&= -2x - \frac{1}{\frac{1}{2}x + 1}\frac{1}{2} \\
&= -2x - \frac{1}{x + 2}
\end{aligned}$$

So, $y'\Big|_{x=0} = -2(0) - \frac{1}{0+2} = -\frac{1}{2}$ and $y\Big|_{0} = 4$.

An equation for the tangent line to the graph of the given equation at $(0, 4)$ is given by

$$y = f(a) + f'(a)(x - a) = 4 - \frac{1}{2}(x - 0) = 4 - \frac{1}{2}x$$

P 2. Let

$$f(x) = \begin{cases} \cos(\pi(x+2)) + 1, & x \le -2 \\ |x+1| + 2, & -2 < x \le 0 \\ \ln(x) - 3, & 0 < x < e \\ \sec(\pi(x-3)), & e \le x \end{cases}$$

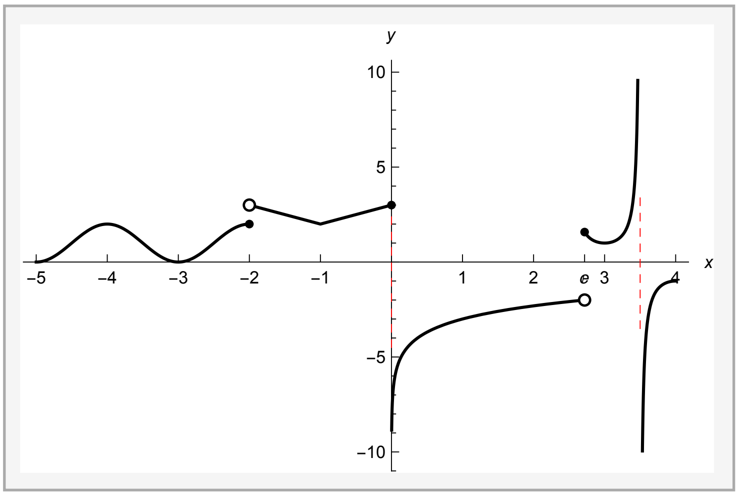
(a) Graph $f(x)$.

(b) Determine all points where $f(x)$ is discontinuous and explain why.

(c) Determine all the points where $f(x)$ is not differentiable and explain why.

Solution:

(a)



(b) f has non-removable discontinuities at $x = -2, 0, e$, and 3.5. Of these $x = 0$ and 3.5 are essential discontinuities and $x = -2$ and e are jump discontinuities. The are no removable discontinuities.

(c) f is not differentiable at $x = -2, 0, e$ and 3.5 since f is not continuous at these x values. In addition, f is not differentiable at $x = -1$, since there the graph of f has a sharp corner.