# Homework 3

Name: **SOLUTIONS** Date: June 2, 2015

**P 1.** Using the basic rules and general properties of derivatives find the derivative of

$$f(x) = \frac{x^2 \cos x}{x^2 e^x + 3} - 7 \ln x + 14 \tan x$$

Show your work!

***Solution***:

$$\begin{aligned}
f'(x) &= \frac{d}{dx}\left[\frac{x^2 \cos x}{x^2 e^x + 3} - 7\ln x + 14\tan x\right] \\
&= \frac{d}{dx}\left[\frac{x^2 \cos x}{x^2 e^x + 3}\right] - \frac{d}{dx}\left[7\ln x\right] + \frac{d}{dx}\left[14\tan x\right] \\
&= \frac{d}{dx}\left[\frac{x^2 \cos x}{x^2 e^x + 3}\right] - 7\frac{d}{dx}\left[\ln x\right] + 14\frac{d}{dx}\left[\tan x\right] \\
&= \frac{d}{dx}\left[\frac{x^2 \cos x}{x^2 e^x + 3}\right] - 7 \cdot \frac{1}{x} + 14\sec^2 x \\
&= \frac{(x^2e^x+3)\frac{d}{dx}\left[x^2\cos x\right] - x^2\cos x \frac{d}{dx}\left[x^2e^x+3\right]}{(x^2e^x+3)^2} - \frac{7}{x} + 14\sec^2 x
\end{aligned}$$

Note that

$$\frac{d}{dx}\left[x^2\cos x\right] = \frac{d}{dx}[x^2] \cdot \cos x + \frac{d}{dx}[\cos x] \cdot x^2 = 2x\cos x - x^2 \sin x$$

and

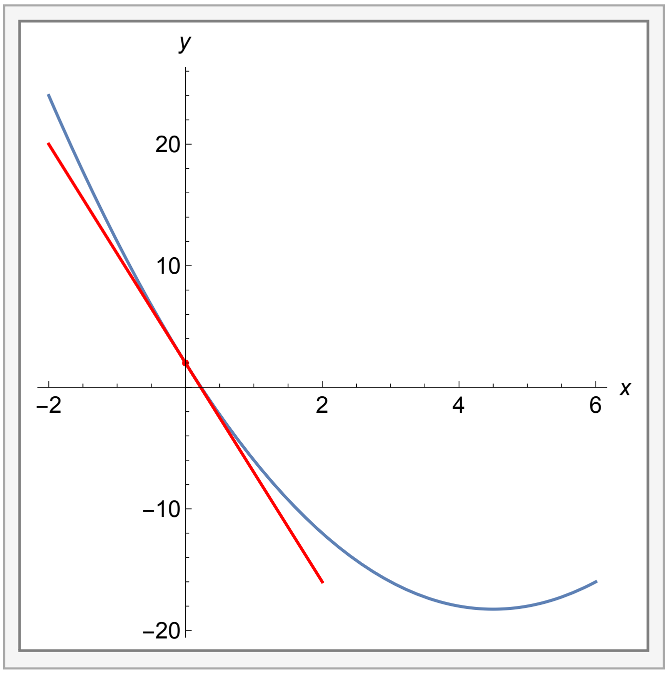
$$\frac{d}{dx}\left[x^2e^x + 3\right] = \frac{d}{dx}[x^2e^x] = 2xe^x + x^2e^x$$

So,

$$f'(x) = \frac{(x^2e^x+3)(2x\cos x - x^2\sin x) - x^2\cos x(2xe^x + x^2e^x)}{(x^2e^x+3)^2} - \frac{7}{x} + 14\sec^2 x$$

**P 2.** Find an equation for the tangent line to the graph of the equation $y = x^2 - 9x + 2$ at $x = 0$.

***Solution***:




We have

$$y' = \frac{d}{dx}[x^2 - 9x + 2] = \frac{d}{dx}[x^2] - \frac{d}{dx}[9x] + \frac{d}{dx}[2] = 2x - 9\frac{d}{dx}[x] = 2x - 9.$$

So,

$$y'\Big|_{x=0} = 2(0) - 9 = -9.$$

The $y$-coordinate of the point at $x = 0$ is

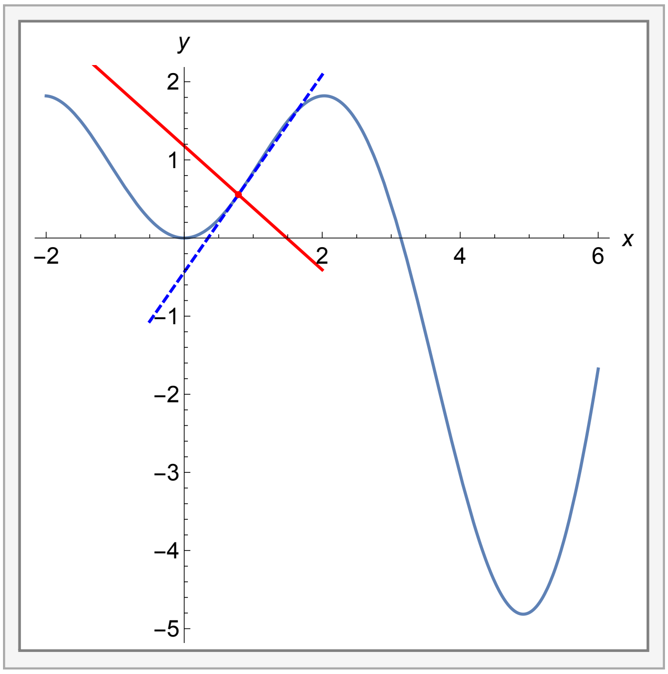
$$y\Big|_{x=0} = 0^2 - 9(0) + 2 = 2.$$

By the point-slope formula, an equation for the tangent line at the point $(0, 2)$ is

$$y - y_1 = m(x - x_1) \Rightarrow y = y_1 + m(x - x_1) \Rightarrow y = 2 - 9(x - 0) = 2 - 9x$$

**P 3.** Find an equation for the perpendicular to the tangent line to the graph of the equation $y = x\sin x$ at $x = \pi/4$.

***Solution***:




We have

$$y' = \frac{d}{dx}[x\sin x] = \frac{d}{dx}[x]\cdot\sin x + \frac{d}{dx}[\sin x]\cdot x = \sin x + x\cos x$$

So,

$$y'\Big|_{x=\pi/4} = \sin\frac{\pi}{4} + \frac{\pi}{4}\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\pi}{4}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} + \frac{\pi\sqrt{2}}{8}$$

The slope of the line perpendicular to the tangent line is

$$\frac{-1}{y'\Big|_{x=\pi/4}} = \frac{-4}{(2+\pi)\sqrt{2}}.$$

The $y$-coordinate of the point at $x = \pi/4$ is

$$y\Big|_{x=\pi/4} = \frac{\pi}{4}\sin\frac{\pi}{4} = \frac{\pi\sqrt{2}}{8}$$

By the point-slope formula, an equation for the tangent line at the point $(0, 2)$ is

$$y - y_1 = m(x - x_1) \Rightarrow y = y_1 + m(x - x_1) \Rightarrow y = \frac{\pi\sqrt{2}}{8} - \frac{8}{(4+\pi)\sqrt{2}}\left(x - \frac{\pi}{4}\right)$$