Homework 2

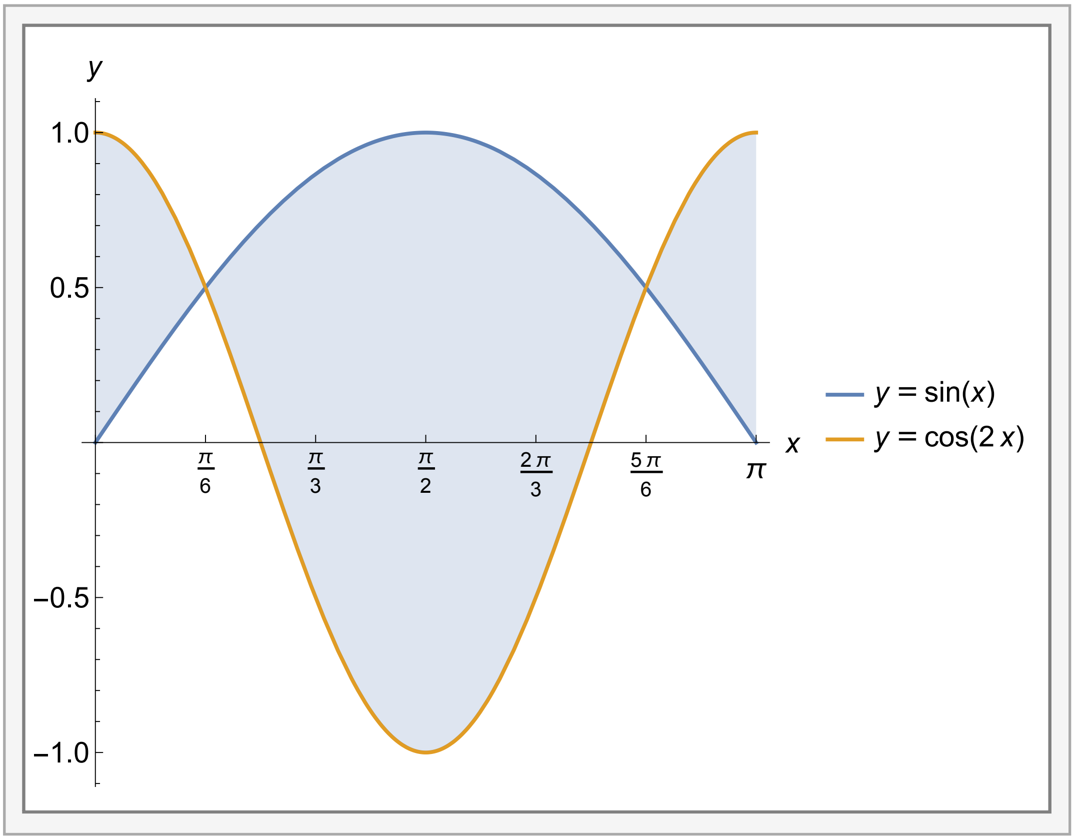
Name: **SOLUTIONS** Date: July 13, 2015

P 1. Sketch the region between the graphs of the equations

$$y = \sin x \text{ and } y = \cos 2x$$

on the interval $[0, \pi]$. Find the area of the region.

Solution:



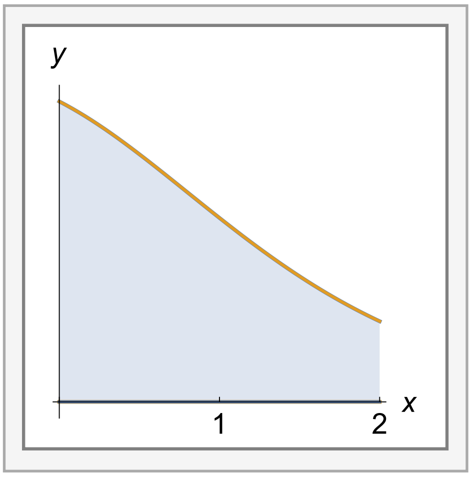
$$\begin{aligned} A &= \int_0^{\pi/6} \cos 2x - \sin x \, dx + \int_{\pi/6}^{5\pi/6} \sin x - \cos 2x + \int_{5\pi/6}^{\pi} \cos 2x - \sin x \, dx \\ &= \left(\frac{\sin 2x}{2} + \cos x \right) \Big|_0^{\pi/6} + \left(-\cos x - \frac{\sin 2x}{2} \right) \Big|_{\pi/6}^{5\pi/6} + \left(\frac{\sin 2x}{2} + \cos x \right) \Big|_{5\pi/6}^{\pi} \\ &= 3\sqrt{3} - 2 \end{aligned}$$

P 2. Find the volume of the solid obtained by revolving the region bounded by the graphs of

$$y = \frac{3^{x/2}}{(1+3^x)^{3/2}}, y = 0, x = 0, x = 2$$

about the x-axis.

Solution: The shaded region, in the figure below, corresponds to the region bounded by the graphs of the given curves.



The volume of the solid of revolution, using the disk method, is given by

$$\int_a^b \pi([f(x)]^2 - [g(x)]^2)\, dx$$

where $f(x) = \frac{3^{x/2}}{(1+3^x)^{3/2}}$ and $g(x) = 0$.

$$\begin{aligned}
\int_a^b \pi([f(x)]^2 - [g(x)]^2)\, dx &= \int_0^2 \pi \left(\frac{3^{x/2}}{(1+3^x)^{3/2}}\right)^2 dx \\
&= \int_0^2 \frac{\pi 3^x}{(1+3^x)^3}\, dx \\
&= \int_2^{10} \frac{\pi}{u^3 \ln 3}\, du \\
&= \int_2^{10} \frac{\pi}{\ln 3} u^{-3}\, du \\
&= \frac{\pi}{\ln 3}\left(\frac{u^{-2}}{-2}\right)\Big|_2^{10} \\
&= \frac{-\pi}{2u^2 \ln 3}\Big|_2^{10} \\
&= \frac{-\pi}{2(10)^2 \ln 3} + \frac{\pi}{8 \ln 3} \approx 0.343152
\end{aligned}$$