# Homework 14

Name: **SOLUTIONS** Date: August 12, 2015

**P 1.** Solve the differential equation.

$$\frac{dy}{dx} = \frac{-7x^2 - 3x - 8}{(x-1)(x+2)\left(x^2+1\right)}\sqrt{y^2+1}$$

***Solution***: The differential equation is separable.

$$\int \frac{1}{\sqrt{y^2+1}}\,dy = \int \frac{-7x^2-3x-8}{(x-1)(x+2)\left(x^2+1\right)}dx$$

$$\begin{aligned}
\int \frac{1}{\sqrt{y^2+1}}\,dy &= \int \frac{1}{\sqrt{(\tan t)^2+1}}\,\sec^2 t\; dt \\
&= \int \sec t\; dt \\
&= \ln|\sec t + \tan t| + C_1 \\
&= \ln|\sqrt{y^2+1} + y| + C_1
\end{aligned}$$

$$\begin{aligned}
\int \frac{-7x^2-3x-8}{(x-1)(x+2)\left(x^2+1\right)} &= \int \frac{x}{x^2+1} - \frac{3}{x-1} + \frac{2}{x+2}\,dx \\
&= \underbrace{\int \frac{x}{x^2+1}\,dx}_{u-sub} - 3\int \frac{1}{x-1}\,dx + 2\int \frac{1}{x+2}\,dx \\
&= \frac{1}{2}\ln|x^2+1| - 3\ln|x-1| + 2\ln|x+2| + C_2
\end{aligned}$$

So, a solution the solution to the differential equation is

$$\ln|\sqrt{y^2+1} + y| = \frac{1}{2}\ln|x^2+1| - 3\ln|x-1| + 2\ln|x+2| + C$$

**P 2.** The number of bacteria in a culture is increasing according to the law of exponential growth. There are 125 bacteria in the culture after 2 hours and 350 bacteria after 4 hours.

(a) Find the initial population.

(b) Write an exponential growth model for the bacteria population. Let $t$ represent time in hours.

(c) Use the model to determine the number of bacteria after 8 hours.

(d) After how many hours will the bacteria count be 25,000?

***Solution***: Let $B$ be the number of bacteria after $t$ hours. Then,

$$B = Ce^{kt}.$$

It is given that $B(2) = 125$ and $B(4) = 350$. So we have the following system of equations:

$$\begin{cases} 125 = Ce^{2k} \\ 350 = Ce^{4k} \end{cases}$$

Dividing the first equation from the second we get:

$$350/125 = e^{4k}/e^{2k} \Rightarrow \frac{14}{5} = e^{2k} \Rightarrow k = \frac{1}{2}\ln\frac{14}{5}$$

Substituting this into the first equation we get:

$$125 = Ce^{\ln\frac{14}{5}} \Rightarrow 125 = C\frac{14}{5} \Rightarrow C = 625/14.$$

So, the exponential model is given by

$$B(t) = \frac{625}{14}e^{\frac{1}{2}t\ln\frac{14}{5}}$$

(a) The initial population is given by $B(0)$ which is equal to $\frac{625}{14} \approx 44.64$

(b) The exponential model is

$$B = \frac{625}{14}e^{\frac{1}{2}t\ln\frac{14}{5}}$$

(c) The number of bacteria after 8 hours is given by $B(8)$ which is equal to

$$B(8) = \frac{625}{14}e^{4\ln\frac{14}{5}} \approx 2744$$

(d) Solving the equation

$$25000 = \frac{625}{14}e^{\frac{1}{2}t\ln\frac{14}{5}}$$

gives us the time $t$ at which the bacteria count reaches 25000. So,

$$t = \frac{2}{\ln\frac{14}{5}}\ln(560) \approx 12.2918$$