Homework 12

Name: **SOLUTIONS** Date: August 12, 2015

P 1. Graph the following piecewise parametric functions on the same coordinate axes.

$$f(t) = \begin{cases} \{-5, 2t\} & -\frac{1}{2} \le t \le \frac{1}{2} \\ \{-3, 2(t-1)\} & \frac{1}{2} \le t \le \frac{3}{2} \\ \{-t - \frac{5}{2}, |t - \frac{3}{2}|\} & \frac{3}{2} \le t \le \frac{5}{2} \\ \{t - \frac{13}{2}, |t - \frac{5}{2}|\} & \frac{5}{2} \le t \le \frac{7}{2} \end{cases}$$

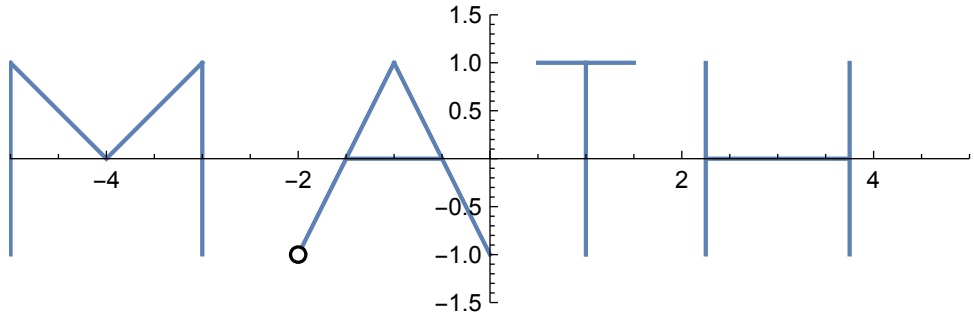
$$g(t) = \begin{cases} \{t-2, 2t-1\} & 0 < t \le 1 \\ \{t-2, 3-2t\} & 1 < t \le 2 \\ \{t - \frac{7}{2}, 0\} & 2 < t \le 3 \end{cases}$$

$$h(t) = \begin{cases} \{t+1, 1\} & -\frac{1}{2} \le t \le \frac{1}{2} \\ \{1, t-4\} & 3 \le t \le 5 \end{cases}$$

$$r(t) = \begin{cases} \{\frac{9}{4}, 2t\} & -\frac{1}{2} \le t \le \frac{1}{2} \\ \{\frac{15}{4}, 2(t-1)\} & \frac{1}{2} \le t \le \frac{3}{2} \\ \{t + \frac{3}{4}, 0\} & \frac{3}{2} < t \le 3 \end{cases}$$

Your final graph should be completely contained in the rectangle defined by $-5 \le x \le 5$ and $-1 \le y \le 1$. [Hint: If the graph is correct, you should be able to recognize it.]

Solution:



P 2. Find the area of the surface of revolution obtained by revolving the curve defined by the parametric equations

$$x = 3\cos\theta,\ y = 3\sin\theta$$

with $0 \le \theta \le \pi/2$ about the $y-$axis.

Solution:

$$\begin{aligned}
S &= \int_a^b 2\pi x\sqrt{(dx/d\theta)^2 + (dy/d\theta)^2}\, d\theta \\
&= \int_0^{\pi/2} 2\pi(3\cos\theta)\sqrt{(-3\sin\theta)^2 + (3\cos\theta)^2}\, d\theta \\
&= 6\pi\int_0^{\pi/2} \cos\theta\sqrt{9(\cos^2\theta + \sin^2\theta)}\, d\theta \\
&= 18\pi\int_0^{\pi/2} \cos\theta\, d\theta \\
&= 18\pi\sin\theta\Big|_0^{\pi/2} \\
&= 18\pi
\end{aligned}$$