Homework 11

Name: **SOLUTIONS** Date: June 18, 2015

P 1. Using the limit definition of the definite integral, find

$$\int_{-2}^{2} 3x^2 + 2x - 1 \, dx$$

Solution:

$$\begin{aligned} A_i &= h \cdot b = f(x_i)\Delta x \\ &= \left(3\left(-2 + i\frac{4}{n}\right)^2 + 2\left(-2 + i\frac{4}{n}\right) - 1\right)\frac{4}{n} \\ &= \frac{192i^2}{n^3} - \frac{160i}{n^2} + \frac{28}{n} \end{aligned}$$

So,

$$\begin{aligned} \sum_{i=1}^{n} A_i &= \sum_{i=1}^{n}\left(\frac{192i^2}{n^3} - \frac{160i}{n^2} + \frac{28}{n}\right) \\ &= \frac{192}{n^3}\sum_{i=1}^{n} i^2 - \frac{160}{n^2}\sum_{i=1}^{n} i + \sum_{i=1}^{n}\frac{28}{n} \\ &= \frac{192}{n^3}\frac{n(n+1)(2n+1)}{6} - \frac{160}{n^2}\frac{n(n+1)}{2} + \frac{28}{n}\cdot n \\ &= \frac{32(n+1)(2n+1)}{n^2} - \frac{80(n+1)}{n} + 28 \end{aligned}$$

and finally,

$$\int_{-2}^{2} 3x^2+2x-1 \, dx = \lim_{n\to\infty}\sum_{i=1}^{n} A_i = \lim_{n\to\infty}\left(\frac{32(n+1)(2n+1)}{n^2} - \frac{80(n+1)}{n} + 28\right) = 64-80+28 = 12$$

Check:

$$\int_{-2}^{2} 3x^2 + 2x - 1 \, dx = \left(x^3 + x^2 - x\right)\Big|_{-2}^{2} = 2^3 + 2^2 - 2 - ((-2)^3 + (-2)^2 - (-2)) = 12$$

P 2. Let

$$f(x) = \begin{cases} |x+3| - 1 & x < -1 \\ 1 & -1 \le x \le 1 \\ 2 - x & 1 \le x \end{cases}$$

Sketch the graph of $f(x)$ and use the graph to answer the following.

(a) $\int_{-2}^{3} f(x)\,dx = 5/2$

(b) $\int_{4}^{0} f(x)\,dx = 1/2$

(c) $\int_{-6}^{-4} 5f(x)\,dx = 10$

(d) $\int_{-6}^{4} f(x)\,dx = 2$

(e) $\int_{-4}^{-2} |f(x)|\,dx = 1$

Solution: