Homework 11

Name: **SOLUTIONS** Date: August 5, 2015

P 1. Find the Maclaurin series for

$$f(x) = \ln(2x+1).$$

Solution:

$$\begin{aligned}
f(x) &= \ln(2x+1) \\
&= \int \frac{2}{2x+1}\, dx \\
&= 2\int \frac{1}{1-(-2x)}\, dx \\
&= 2\int \sum_{n=0}^{\infty} (-2x)^n\, dx \\
&= 2\sum_{n=0}^{\infty} (-1)^n 2^n \frac{x^{n+1}}{n+1} + C \\
&= \sum_{n=0}^{\infty} (-1)^n 2^{n+1} \frac{x^{n+1}}{n+1} + C \\
&= \sum_{n=1}^{\infty} (-1)^{n+1} 2^n \frac{x^n}{n} + C
\end{aligned}$$

If $x = 0$ then

$$\ln(2(0)+1) = \sum_{n=1}^{\infty} (-1)^{n+1} 2^n \frac{0^n}{n} + C \Leftrightarrow 0 = 0 + C \Leftrightarrow C = 0.$$

So,

$$\ln(2x+1) = \sum_{n=1}^{\infty} (-1)^{n+1} 2^n \frac{x^n}{n}.$$

P 2. Find the Taylor series for

$$f(x) = e^x$$

centered at 1.

Solution:

$$e^x = e^{x-1+1} = e^{x-1}e = \sum_{n=0}^{\infty} \frac{x^n}{n!}e = \sum_{n=0}^{\infty} \frac{e \cdot x^n}{n!}.$$