Homework 10

Name: **SOLUTIONS** Date: June 16, 2015

P 1. Find
$$\int \sin x + 3\cos x - \frac{23}{x^2+1} - \frac{5x^2+3x+1}{\sqrt[5]{x^2}}\, dx$$

Solution:

$$\begin{aligned}
&\int \sin x + 3\cos x - \frac{23}{x^2+1} - \frac{5x^2+3x+1}{\sqrt[5]{x^2}}\, dx \\
&\qquad = \int \sin x\, dx + \int 3\cos x\, dx - \int \frac{23}{x^2+1}\, dx - \int \frac{5x^2+3x+1}{\sqrt[5]{x^2}}\, dx \\
&\qquad = -\cos x + 3\int \cos x\, dx - 23\int \frac{1}{x^2+1}\, dx - \int \frac{5x^2+3x+1}{\sqrt[5]{x^2}}\, dx \\
&\qquad = -\cos x + 3\sin x - 23\arctan x - \int \frac{5x^2+3x+1}{\sqrt[5]{x^2}}\, dx \\
&\qquad = -\cos x + 3\sin x - 23\arctan x - \int 5x^{8/5} + 3x^{3/5} + x^{-2/5}\, dx \\
&\qquad = -\cos x + 3\sin x - 23\arctan x - \left(\frac{25}{13}x^{13/5} + \frac{15}{8}x^{8/5} + \frac{5}{3}x^{3/5}\right) + C
\end{aligned}$$

P 2. Using summation formulas, find the area of the region bounded by the graph of $y = x^2 + 2$ on the interval $[0, 1]$.

Solution: If we use n rectangles, then the base of each rectangle is $\Delta x = (b - a)/n = 1/n$. If we want the height of the rectangle to be given by the function at the left endpoint of each interval, then we should start at $x = 0$. To get the height of the next rectangle, we evaluate at $0 + \Delta x$. To find the height of the ith rectangle we evaluate at $0 + i\Delta x$. The area of the ith rectangle is given by $f(0 + i\Delta x)\Delta x$. To get our approximation we sum the areas of all the rectangles and denoted this sum by A_n. So,

$$\begin{aligned}
A_n &= \sum_{i=1}^{n} f(0 + i\Delta x)\Delta x \\
&= \sum_{i=1}^{n} ((i\Delta x)^2 + 2)\Delta x \\
&= \sum_{i=1}^{n} ((i(1/n))^2 + 2)(1/n) \\
&= \sum_{i=1}^{n} \left(\frac{i^2}{n^3} + \frac{2}{n}\right) \\
&= \sum_{i=1}^{n} \frac{i^2}{n^3} + \sum_{i=1}^{n} \frac{2}{n} \\
&= \frac{1}{n^3} \sum_{i=1}^{n} i^2 + \frac{2}{n} \cdot n \\
&= \frac{1}{n^3} \frac{n(n+1)(2n+1)}{6} + 2 \\
&= \frac{(n+1)(2n+1)}{6n^2} + 2
\end{aligned}$$

The exact area of the bounded region is

$$A = \lim_{n\to\infty} A_n = \lim_{n\to\infty} \left(\frac{(n+1)(2n+1)}{6n^2} + 2\right) = 2 + \frac{1}{3} = 7/3$$