Homework 1

Name: **SOLUTIONS** Date: May 28, 2015

P 1. Let

$$f(x) = \begin{cases} \sin(\pi(x+3)), & x \leq -3 \\ e^{x+2} - e^{-1}, & -3 < x \leq -1 \\ \ln(x+1), & -1 < x < -1+e \\ \tan(\pi(x-2)), & -1+e \leq x \end{cases}$$

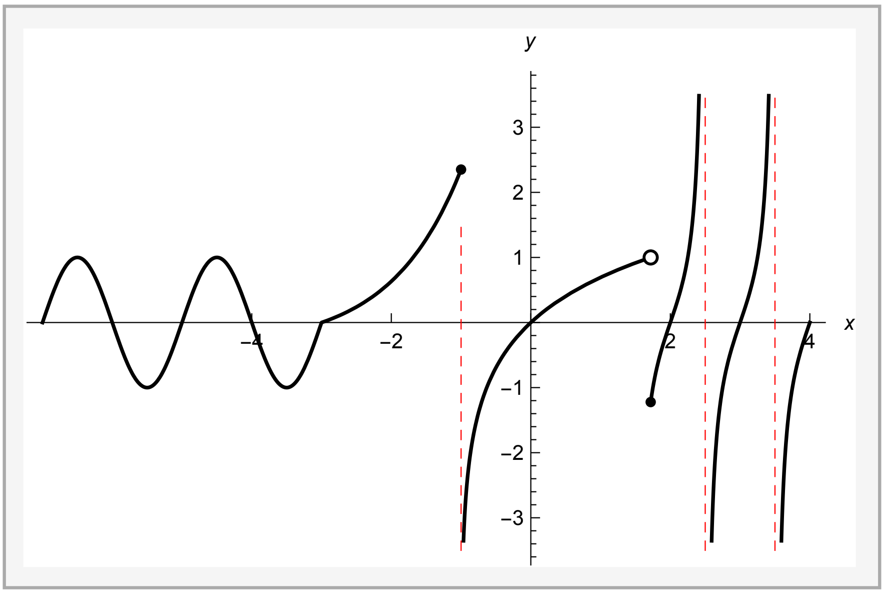
(a) Complete the table.

x	-3	-1.01	-1.001	-1	-0.999	-0.99	-0.9	$e-1$
$f(x)$								

Solution:

x	-3	-1.01	-1.001	-1	-0.999	-0.99	-0.9	$e-1$
$f(x)$	0	2.32336	2.34769	2.3504	-6.90776	-4.60517	-2.30259	-1.22216

(b) Graph f. ***Solution***:



(c) Find

(i) $\lim_{x \to -3^+} f(x)$

Solution:

$$\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} (e^{x+2} - e^{-1}) = e^{-1} - e^{-1} = 0$$

(ii) $\lim_{x \to -3^-} f(x)$

Solution:

$$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \sin(\pi(x+3)) = \sin(\pi(-3+3)) = 0$$

(iii) $\lim_{x \to -3} f(x)$

Solution: From (i) and (ii), we have that

$$\lim_{x \to -3} f(x) = 0.$$

(iv) $\lim_{x \to -2} f(x)$

Solution:

$$\lim_{x \to -2} f(x) = \lim_{x \to -2^-} (e^{x+2} - e^{-1}) = e^{-2+2} - e^{-1} = 1 - e^{-1}$$

(v) $f(100)$

Solution:

$$f(100) = \tan(\pi(100-2)) = \tan(98\pi) = \tan 0 = 0$$

(vi) $\lim_{x \to -1} f(x)$

Solution:

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (e^{x+2} - e^{-1}) = e^{1} - e^{-1}$$

and

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \ln(x+1) = -\infty$$

Since the limits from the right and left of one do not agree, the limit at -1 does not exist. That is,

$$\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x) \Rightarrow \lim_{x \to -1} f(x) \text{ DNE}$$

P 2. Let a be the negative solution to the equation

$$x^3 - 7x^2 + 7x + 15 = 0.$$

Find

$$\lim_{x \to a} \frac{|x-a|}{x-a}$$

[Hint: $f(x) = x^3 - 7x^2 + 7x + 15$ has a zero $x = 5$.]

Solution: Since $x^3-7x^2+7x+15 = 0$ has a zero $x = 5$, then $x-5$ is a factor of $x^3-7x^2+7x+15$. By long division or synthetic division,

$$x^3 - 7x^2 + 7x + 15 = (x-5)(x^2 - 2x - 3) = (x-5)(x-3)(x+1).$$

Therefore, $x^3 - 7x^2 + 7x + 15 = 0$ has solutions $x = 5, 3$, and -1. So $a = -1$.

Now,

$$\lim_{x \to a} \frac{|x-a|}{x-a} = \lim_{x \to -1} \frac{|x+1|}{x+1}$$

(i)

$$\lim_{x \to -1^+} \frac{|x+1|}{x+1} = \lim_{x \to -1^+} \frac{x+1}{x+1} = 1$$

(ii)

$$\lim_{x \to -1^-} \frac{|x+1|}{x+1} = \lim_{x \to -1^-} \frac{-(x+1)}{x+1} = -1$$

So,

$$\lim_{x \to -1} \frac{|x+1|}{x+1} DNE$$