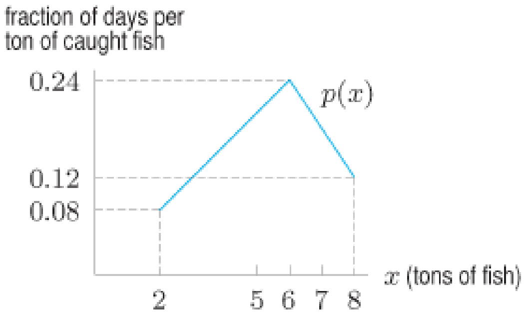
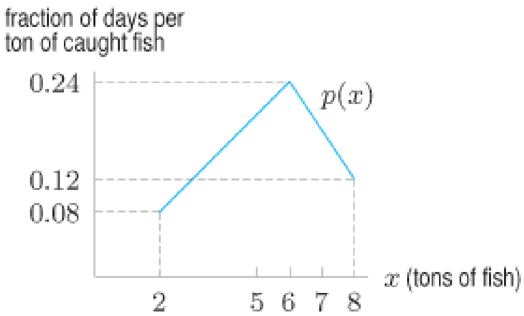
8.8 Probability, Mean, and Median

Name: Date:

P 1. Show that the area under the fishing density function in the figure is 1. Why is this to be expected?



P 2. Find the mean daily catch for the fishing data in the figure.



P 5. A quantity x has cumulative distribution function $P(x) = x - x^2/4$ for $0 \leq x \leq 2$ and $P(x) = 0$ for $x < 0$ and $P(x) = 1$ for $x > 2$. Find the mean and median of x.

P 7. Suppose that x measures the time (in hours) it takes for a student to complete an exam. All students are done within two hours and the density function for x is

$$p(x) = \begin{cases} x^3/4, & \text{if } 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) What proportion of students take between 1.5 and 2.0 hours to finish the exam?

(b) What is the mean time for students to complete the exam?

(c) Compute the median of this distribution.

P 8. In 1950 an experiment was done observing the time gaps between successive cars on the Arroyo Seco Freeway. The data show that the density function of these time gaps was given approximately by

$$p(x) = ae^{-0.122x}$$

where x is the time in seconds and a is a constant.

(a) Find a.

(b) Find P, the cumulative distribution function.

(c) Find the median and mean time gap.

(d) Sketch rough graphs of p and P.

P 9. Consider a group of people who have received treatment for a disease such as cancer. Let t be the *survival time*, the number of years a person lives after receiving treatment. The density function giving the distribution of t is

$$p(t) = Ce^{-Ct}$$

for some positive constant C.

(a) What is the practical meaning for the cumulative distribution function

$$P(t) = \int_0^t p(x)\, dx?$$

(b) The survival function, $S(t)$, is the probability that a randomly selected person survives for at least t years. Find $S(t)$.

(c) Suppose a patient has a 70% probability of surviving at least two years. Find C.

P 15. Which of the following functions makes the most sense as a model for the probability density representing the time (in minutes, starting from $t = 0$) that the next customer walks into a store?

(a) $p(t) = \begin{cases} \cos t, & 0 \leq t \leq 2\pi \\ e^{t-2\pi}, & t \geq 2\pi \end{cases}$

(b) $p(t) = 3e^{-3t}$ for $t \geq 0$

(c) $p(t) = e^{-3t}$ for $t \geq 0$

(d) $p(t) = \frac{1}{4}$ for $0 \leq t \leq 4$