Exam 2 Review Problems

Name: Date:

P 1. Find dy/dx for the following:

(a) $y = 2x\sqrt{x^2+1}$

(b) $y = \sin(\ln(x^2 + 2e^{3x}))$

P 2. Show that

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}.$$

P 3. Find dy/dx for the following:

(a) $y = x^x$

(b) $y = 5^{\sqrt{x^2-2}}$

P 4. Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 3x + 1$ on $[0, 3]$.

P 5. For $f(x) = \dfrac{x}{x^2+9}$ find the following:

(a) Intervals at which $f(x)$ is increasing and intervals on which $f(x)$ is decreasing.

(b) Local maximum and minimum values.

(c) Intervals of concavity and inflection points.

P 6. Show that out of all rectangles with a given fixed area a, the one with the smallest perimeter is a square.

P 7. If $y = u^3 - 1, u = -\frac{2}{v}$, and $v = x^3$; find dy/dx.

P 8. Show that

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}.$$

P 9. Find an equation of the tangent line to the curve $\arctan y = x^2 + y^2$ at the point $(1, 0)$.

P 10. Find $f'(1)$ for $f(x) = 2^{\sqrt{x^2+3}}$

P 11. Find dy/dx in terms of x for $y = x^{\ln x}$.

P 12. Find the area of the largest rectangle with its base on the x-axis and opposite side inscribed in the parabola $y = 4 - x^2$.

P 13. For $f(x) = \dfrac{x^2}{x^2 - 4}$ find the following:

(a) Intervals on which $f(x)$ is increasing or decreasing.

(b) Local maximum and minimum values.

(c) Intervals of concavity and inflection points.

(d) Vertical and horizontal asymptotes.

(e) Sketch of $f(x)$.

P 14. For the following functions, determine the derivative $f'(x)$.

(a) $f(x) = \dfrac{e^{3x}}{1 + \ln x}$

(b) $f(x) = \sin^3(1/x)$

P 15. Find the maximum and minimum values of $y = x^3 - 2x^2 - 4x + 2$ on the interval $[0, 3]$.

P 16. Find an equation of the tangent line to the curve $\cos y - \sin x \sin y = 0$, at the point $(\pi, \pi/2)$.

P 17. For $f(x) = (x^2 - 4)^3$ determine the following:

- the roots of f and the intervals where f is positive or negative;
- the critical points of f and the intervals where f is increasing or decreasing;
- use this information to sketch the curve $y = f(x)$ on the interval $-3 \leq x \leq 3$.

P 18. Find an equation of the tangent line to the curve $\sqrt{x} + \sqrt{y} = 3$ at the point $(4, 1)$.

P 19. A rectangle is inscribed inside an isosceles right triangle whose hypotenuse is of length 2. One side of the rectangle lies on the side of the isosceles triangle of length 2. The other two vertices of the rectangle lie on the equal sides of the isosceles triangle. What is the largest area possible for such a rectangle?

P 20. For the function $f(x) = e^{2x-x^2}$

(a) determine the intervals where f is increasing or decreasing.

(b) determine the intervals where f is concave up or concave down.

(c) identify all local minima, local maxima, and inflection points of f.

(d) sketch the curve $y = f(x)$.

P 21. Find dy/dx for the following functions:

(a) $y = 2x\sqrt{x^2+1}$ (b) $y = x^{\cos x}$ (c) $y = \sin(\arctan(x^2))$

P 22. A patient's temperature change T due to a dose D of a certain drug is given by

$$T = \left(\frac{C}{2} - \frac{D}{3}\right) D^2$$

where C is a positive constant.

(i) What dose maximizes the temperature change?

(ii) The sensitivity of the patient to the drug is defined as dT/dD. What does maximizes the sensitivity?

P 23. A cone shaped coffee filter of radius 6 cm and depth 10 cm is filled with water, which drips out the bottom at a constant rate of $3/2\text{cm}^2$ per second. How fast is the water level falling when the depth is 5 cm?

P 24. For $f(x) = e^{-x^2/2}$ find the following:

(a) Vertical and horizontal asymptotes.

(b) Intervals on which $f(x)$ is increasing or decreasing.

(c) Local maximum and minimum values.

(d) Intervals of concavity and inflection points.

(e) Sketch the graph of f.

P 25. Find the point on the line $y = 3x + 2$ that is closest to $(3, 1)$.

P 26. Find the absolute maximum and absolute minimum values of $f(x) = \arctan\sqrt{x^2+1}$ on the interval $[-\sqrt{2}, \sqrt{2}]$.

P 27. Find y' for the following:

(a) $x \cos y = y$

(b) $y = (x^2 + 3)^{\ln x}$

P 28. Find $f'(0)$ for the following:

(a) $f(x) = e^{mx} \cos nx$

(b) $f(x) = \pi^{\sin x}$