Exam 1 Review Problems

Name: Date:

P 1. Consider

$$f(x) = \begin{cases} Ax^2 - 2x, \text{ if } x < -2 \\ 3, \text{ if } x = -2 \\ x^3 + 2B, \text{ if } x > -2 \end{cases}$$

(a) Find the values of A and B that make $f(x)$ continuous for all real x.

(b) Find the equation of the tangent line to $f(x)$ at $x = 1$.

P 2.

(a) Use the definition of the derivative to find the derivative of $f(x) = C$ where C is a constant.

(b) The line $y = 5x + 1$ is tangent to the curve $f(x) = ax^3 + bx^2 + cx$ at the point $(1, 6)$. Moreover, $f''(1) = -6$. Find a, b, and c.

P 3. Using the definition of the derivative, find the derivative of

$$f(x) = \frac{1}{\sqrt{x-6}}$$

P 4. Find the following limits

(a) $\lim\limits_{x \to 2^-} \dfrac{x^2 - 4}{|x - 2|}$

(b) $\lim\limits_{x \to \infty} (\sqrt{3x^2 - 4x + 2} - \sqrt{3x^2 + 1})$

P 5.

(a) Consider the function

$$f(x) = \begin{cases} \frac{x^2-x-2}{2x+2}, & x \neq -1 \\ A, & x = -1 \end{cases}$$

If possible, choose A such that $f(x)$ is continuous at $x = -1$, or explain why this is not possible. (Justify your answer).

(b) Evaluate $\displaystyle\lim_{x \to 2^-} \frac{2x^2 - 8}{|x-2|}$

P 6. Find the vertical and horizontal asymptotes for the function $f(x) = \dfrac{2e^x}{e^x - 5}$.

P 7. Using the limit definition of the derivative, show that

$$\frac{d}{dx}[2x^2] = 4x.$$

P 8. Find $\lim\limits_{x \to 4^+} \dfrac{4 - x}{|4 - x|}$

P 9. Find $\lim\limits_{x \to 1} \left(\dfrac{1}{x - 1} + \dfrac{1}{x^2 - 3x + 2} \right).$

P 10. Let

$$f(x) = \begin{cases} \sqrt{-x}, & \text{if } x < 0 \\ 3 - x, & \text{if } 0 \le x < 3 \\ (x-3)^2, & \text{if } x > 3 \end{cases}$$

Where is this function discontinuous and why?

P 11. Determine the parabola $y = ax^2 + bx + c$ that passes through the point $(1, 4)$ and whose tangent lines at $x = -1$ and $x = 5$ have slopes 6 and -2, respectively.

P 12. Determine all the horizontal asymptotes for the function

$$f(x) = \frac{x+2}{\sqrt{9x^2+4}}.$$

P 13. Find the following limits. If the limit does not exist, explain why.

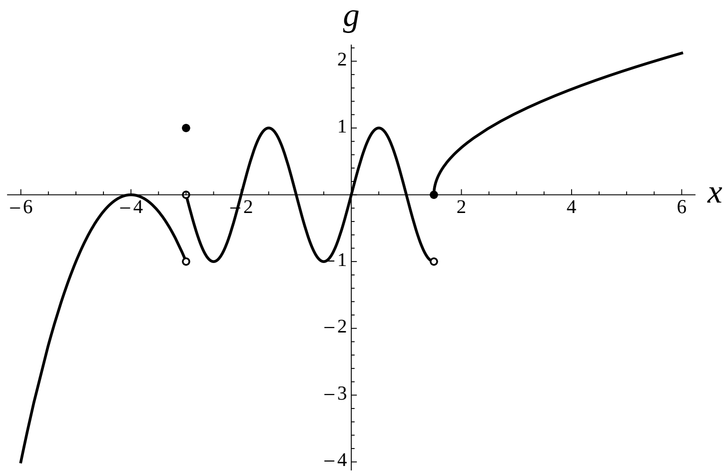
1. $\displaystyle\lim_{x\to 2} \frac{x^2+x-6}{x^2+2x-8}$

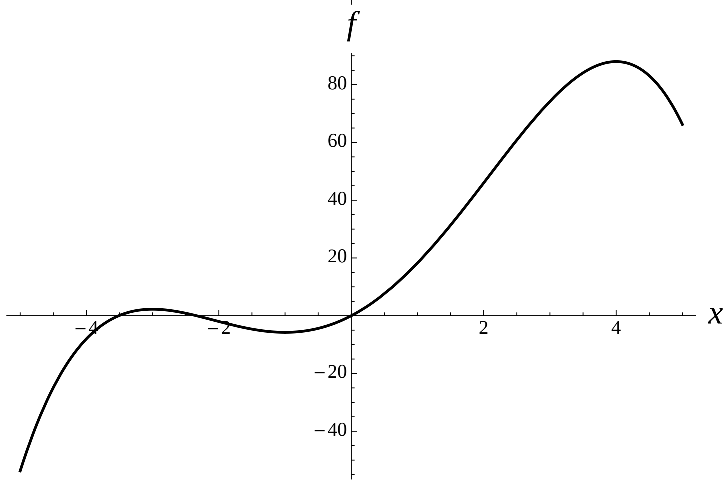
2. $\displaystyle\lim_{x\to 1^+} \frac{-(x^2-1)}{|1-x|}$

P 14. Determine the value of c such that

$$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{cx^2} = 1.$$

P 15. Use the graphs of f and g to answer the following.





(a) Determine all values of x for which g is not differentiable and explain why.

(b) Determine the intervals on which f is concave up.

(c) Determine the inflection points of g.

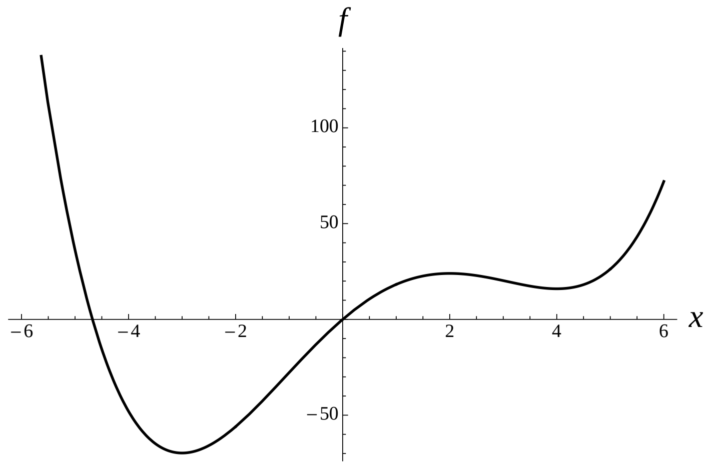
(d) Determine the intervals on which f is increasing.

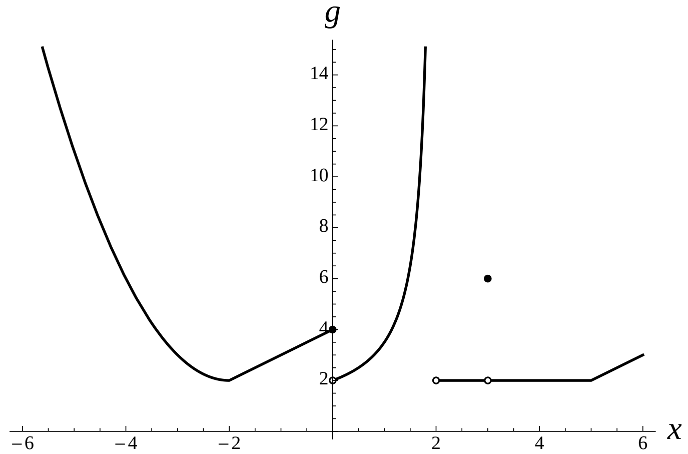
(e) Determine the relative extrema of f.

(f) Does f have a global minimum? If so, what is the global minimum? If not, explain why.

(g) Does f defined on $[-3, 0]$ have a global minimum? If so, what is the global minimum? If not, explain why.

P 16 (21 Points)**.** Use the graphs of f and g to answer the following.





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(e) Determine the relative extrema of f.

(f) Does f have a global maximum? If so, what is the global maximum? If not, explain why.

(g) Does f defined on $[-3, 0]$ have a global maximum? If so, what is the global maximum? If not, explain why.

(h) Determine the intervals on which g is constant.