

16.6 Applications of Integration to Probability

Name:

Date:

P 1. Check whether $p(x, y) = 1/2$ is a joint density function, where $R = \{(x, y) \mid 4 \leq x \leq 5, -2 \leq y \leq 0\}$. Assume $p(x, y) = 0$ outside the region R .

P 3. Check whether $p(x, y) = x + y$ is a joint density function, where $R = \{(x, y) \mid -1 \leq x \leq 1, 0 \leq y \leq 1\}$. Assume $p(x, y) = 0$ outside the region R .

P 5. Check whether $p(x, y) = \frac{2}{\pi}(1 - x^2 - y^2)$ is a joint density function, where $R = \{(x, y) \mid x^2 + y^2 \leq 1\}$. Assume $p(x, y) = 0$ outside the region R .

P 11. Let p be the joint density function such that $p(x, y) = xy$ in R , the rectangle $0 \leq x \leq 2$, $0 \leq y \leq 1$, and $p(x, y) = 0$ outside R . Find the fraction of the population satisfying $x \geq y$.

P 17. A joint density function is given by

$$f(x, y) = \begin{cases} kx^2, & \text{for } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 1, \\ 0, & \text{otherwise} \end{cases}$$

- (a) Find the value of k .
- (b) Find the probability that (x, y) satisfies $x + y \leq 2$.
- (c) Find the probability that (x, y) satisfies $x \leq 1$ and $y \leq 1/2$.