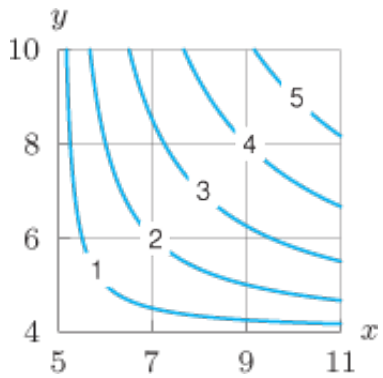
16.1 The Definite Integral of a Function of Two Variables

Name: Date:

P 1. The table below gives values of the function $f(x, y)$, which is increasing in x and decreasing in y on the region $R = \{(x, y) \mid 0 \le x \le 6,\ 0 \le y \le 1\}$. Make the best possible upper and lower estimates of $\int_R f(x, y)\, dA$.

y \ x	0	3	6
0	5	7	10
0.5	4	5	7
1	3	4	6

P 3. The figure below shows contours of $g(x, y)$ on the region R, with $5 \le x \le 11$ and $4 \le y \le 10$. Using $\Delta x = \Delta y = 2$, find an overestimate and an underestimate for $\int_R g(x, y)\, dA$.



P 6. Decide (without calculation) whether the

$$\int_D dA$$

is positive, negative, or zero, where D is the region inside the unit circle centered at the origin.

P 7. Decide (without calculation) whether the

$$\int_R 5x \; dA$$

is positive, negative, or zero, where R is the right half of the unit circle centered at the origin.

P 8. Decide (without calculation) whether the

$$\int_B 5x \; dA$$

is positive, negative, or zero, where B is the bottom half of the unit circle centered at the origin.

P 9. Decide (without calculation) whether the

$$\int_R y^3 + y^5 \, dA$$

is positive, negative, or zero, where D is the unit circle centered at the origin.

P 17. Explain what is wrong with the statement: "For all f, the integral $\int_R f(x, y) \, dA$ gives the volume of the solid under the graph of f over the region R."

P 18. Explain what is wrong with the statement: "If R is a region in the third quadrant where $x < 0$ and $y < 0$, then $\int_R f(x, y) \, dA$ is negative."