14.6 The Chain Rule

Name:

P 3. Let
$$z = \sin(x/y),$$
where $x = 2t$ and $y = 1 - t^2$. Find dz/dt.

P 9. Let
$$z = xe^y,$$
where $x = \ln u$ and $y = v$. Find $\partial z/\partial u$ and $\partial z/\partial v$.

P 11. Let
$$z = xe^y,$$
where $x = u^2 + v^2$ and $y = u^2 - v^2$. Find $\partial z/\partial u$ and $\partial z/\partial v$.

P 13. Let
$$z = xe^{-y} + ye^{-x},$$
where $x = u \sin v$ and $y = v \cos u$. Find $\partial z/\partial u$ and $\partial z/\partial v$.

P 14. Let
$$z = \cos(x^2 + y^2),$$
where $x = u \cos v$ and $y = u \sin v$. Find $\partial z/\partial u$ and $\partial z/\partial v$.

P 15. Let
$$z = \tan^{-1}(x/y),$$
where $x = u^2 + v^2$ and $y = u^2 - v^2$. Find $\partial z/\partial u$ and $\partial z/\partial v$.

P 18. A bison is charging across the plain one morning. His path takes him to location (x, y) at time t where x and y are functions of t and north is in the direction of increasing y. The temperature is always colder farther north. As time passes, the sun rises in the sky, sending out more heat, and a cold front blows in from the east. At time t the air temperature H near the bison is given by $H = f(x, y, t)$. The chain rule expresses the derivative dH/dt as a sum of three terms:

$$\frac{dH}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial t}.$$

Identify the term that gives the contribution to the change in temperature experienced by the bison that is due to

(a) The rising sun.

(b) The coming cold front.

(c) The bison's change in latitude.

P 23. Corn production, C, is a function of rainfall, R, and temperature, T. The figures below show how rainfall and temperature are predicted to vary with time because of global warming. Suppose we know that $\Delta C \approx 3.3\Delta R - 5\Delta T$. Use this to estimate the change in corn production between the year 2020 and the year 2021. Hence, estimate dC/dt when $t = 2020$.
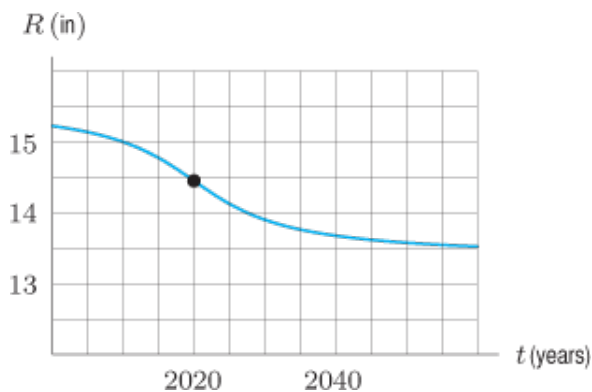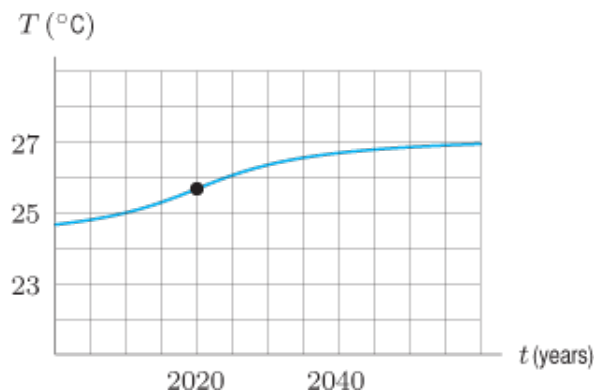

Figure 1: Rainfall as a function of time

Figure 2: Temperature as a function of time

P 35. Let $z = f(x, y)$, $x = x(u, v)$, $y = y(u, v)$, $x(4, 5) = 2$, and $y(4, 5) = 3$. Calculate $z_u(4, 5)$ in terms of $a, b, c, d, e, k, p, q, r, s, t$.

$$\begin{array}{llll}
f_x(4, 5) = a & f_y(4, 5) = c & x_u(4, 5) = e & y_u(4, 5) = p \\
f_x(2, 3) = b & f_y(2, 3) = d & x_v(4, 5) = k & y_v(4, 5) = q \\
x_u(2, 3) = r & y_u(2, 3) = s & x_v(2, 3) = t & y_v(2, 3) = w
\end{array}$$