14.5 Gradients and Directional Derivatives in Space

Name: Date:

P 7. Find the gradient of $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.

P 16. Find the gradient of $f(x, y, z) = xyz$, at $(1, 2, 3)$.

P 27. Find a normal vector to the surface given by the equation $y^2 = z^2 - 3$ and an equation for the tangent plane to the surface at $(-1, 1, 2)$.

P 40. Find an equation of the tangent plane to the surface given by $x^2 + y^2 = 1$ at the point $(1, 0, 0)$.

P 54. Your house lies on the surface $z = f(x, y) = 2x^2 - y^2$ directly above the point $(4, 3)$ in the xy-plane.

(a) How high above the xy-plane do you live?

(b) What is the slope of your lawn as you look from your house directly toward the z-axis (that is, along the vector $-4\vec{i} - 3\vec{j}$)?

(c) When you wash your car in the driveway, on this surface above the point $(4, 3)$, which way does the water run off? (Give your answer as a two-dimensional vector.)

(d) What is the equation of the tangent plane to this surface at your house?

P 55.

(a) Sketch the contours of $z = y - \sin x$ for $z = -1, 0, 1, 2$.

(b) A bug starts on the surface at the point $(\pi/2, 1, 0)$ and walks on the surface $z = y - \sin x$ in the direction parallel to the y-axis, in the direction of increasing y. Is the bug walking in a valley or on top of a ridge? Explain.

(c) On the contour $z = 0$ in your sketch for part a, draw the gradients of z at $x = 0$, $x = \pi/2$, and $x = \pi$.