10.4 The Error in Taylor Polynomial Approximations

Name: Date:

P 4. Use the following theorem to find a bound for the error in approximating $\sqrt{0.9}$ with a third-degree Taylor polynomial for $f(x) = \sqrt{1+x}$ about $x = 0$.

T 1. *Suppose f and all its derivatives are continuous. If $P_n(x)$ is the n^{th} Taylor polynomial for $f(x)$ about a, then*

$$|E_n(x)| = |f(x) - P_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1},$$

where $|f^{n+1}| \le M$ on the interval between a and x.

P 6. Use the following theorem to find a bound for the error in approximating $1/\sqrt{3}$ with a third-degree Taylor polynomial for $f(x) = (1+x)^{-1/2}$ about $x = 0$.

T 2. *Suppose f and all its derivatives are continuous. If $P_n(x)$ is the n^{th} Taylor polynomial for $f(x)$ about a, then*

$$|E_n(x)| = |f(x) - P_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1},$$

where $|f^{n+1}| \leq M$ on the interval between a and x.

P 13. Consider the error in using the approximation $\sin\theta \approx \theta$ on the interval $[-1, 1]$.

(a) Reasoning informally, say where the approximation is an overestimate and where it is an underestimate.

(b) Use the following theorem to bound the error. Check your answer graphically.

T 3. *Suppose f and all its derivatives are continuous. If $P_n(x)$ is the n^{th} Taylor polynomial for $f(x)$ about a, then*

$$|E_n(x)| = |f(x) - P_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1},$$

where $|f^{n+1}| \leq M$ on the interval between a and x.

P 14. Repeat problem 13 for the approximation $\sin\theta \approx \theta - \theta^3/3!$.

P 18. Give a bound for the error for the n^{th}-degree Taylor polynomial about $x = 0$ approximating $\cos x$ on the interval $[0, 1]$. What is the bound for $\sin x$?

P 19. What degree Taylor polynomial about $x = 0$ do you need to calculate $\cos 1$ to four decimal places? To six decimal places? Justify your naswer using the results of Problem 18.

P 21. Show that the Taylor series about 0 for e^x converges to e^x for every x. Do this by showing that the error $E_n(x) \to 0$ as $n \to \infty$.