Exam 1 Review Problems

Name: Date:

P 1. Find the solution to the initial value problem

$$\frac{dy}{dx} = x^2 + x^3, \quad y(1) = 1$$

P 2. Use the given table to approximate $\int_0^{16} f(x)\, dx$

x	0	4	8	12	16
$f(x)$	32	22	15	11	9

P 3. Evaluate the following integrals:

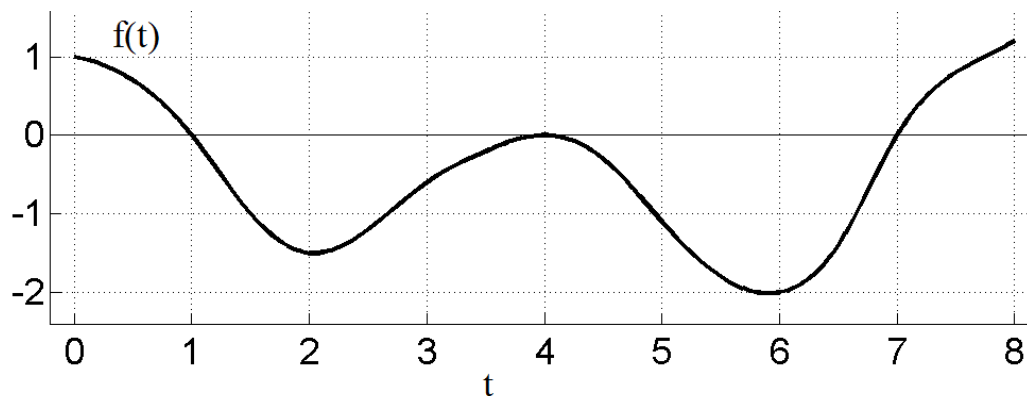(a) $\int t\sqrt{t^2 - \ln 2}\, dt$

(b) $\int x \cos kx\, dt$

P 4. Evaluate

$$\int_0^{3/2} \sin \pi t \cos \pi t\, dt$$

P 5. Let $= \int_0^x f(t)\, dt$, where f is the function whose graph is shown.

(a) What are the critical points of $g(x)$?

(b) For each critical point of $g(x)$, indicate if it is a local max, local min, or neither.

(c) Determine the intervals for which $g(x)$ is concave up or concave down. What are the inflection points?

P 6. Set up an integral (but **do not evaluate**) that represents the area enclosed by the given curves.

(a) $y = |x|$ and $y = x^2 - 2$

(b) $y = 2x$ and $x = 1 - (y - 1)^2$

P 7. When a particle is located a distance x meters from the origin, a force of $3x^2$ newtons acts on it and moves it towards the right.

(a) Write down a sum which approximates the word done by the force as the particle is moved from $x = 0$ to $x = 3$.

(b) Find the exact amount of work done from $x = 0$ to $x = 3$.

(c) At what point x on the interval $0 < x < 3$ is the force the same as its average over the entire interval?

P 8. Find the following antiderivatives:

(a) $\displaystyle\int \sin x \cos^2 x \, dx$

(b) $\displaystyle\int e^x \sqrt{e^x - 1} \, dx$

P 9. Find the following antiderivatives:

(a) $\displaystyle\int x e^{2x} \, dx$

(b) $\displaystyle\int x^{3/2} \ln x \, dx$

P 10. Find the exact area of the region that lies beneath the curve $f(x) = x\sqrt{a^2 - x^2}$ and above the x-axis for $0 \leq x \leq a$ (a is a constant).

P 11. Find

$$\int \sin^3(2x) \, dx$$

P 12. A ball is thrown from the top of a building straight up from the ledge with initial velocity of 16 ft/s. If the height of the building is 96 feet and we know that constant deceleration due to gravity is -32 ft/s^2, answer the following question:

(a) When does the ball hit the ground?

(b) What is the ball's velocity when it hits the ground?

(c) What is the greatest height the ball reaches?

P 13. Find

(a) $\displaystyle\int \cos x \sin^6 x \, dx$

(b) $\displaystyle\int_0^{\frac{2\pi}{3}} \tan x \, dx$

P 14. Find

(a) $\displaystyle\int x^2 e^x \, dx$

(b) $\displaystyle\int_1^2 \frac{\ln x}{x^2} \, dx$

P 15. Let $f(x) = (\ln x)^2$ and $g(x) = x(\ln x)^2 - 2x \ln x + 2x$. Verify that $g(x)$ is an antiderivative of $f(x)$.
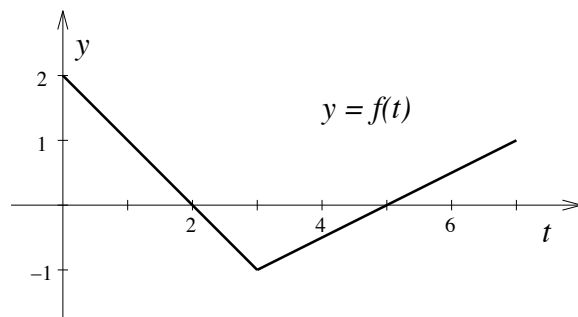
P 16. A particle moves along a line with velocity at time t given by $v(t) = t^2 - t - 6$ (measured in meters per second). Find the distance travelled during the interval $1 \leq t \leq 4$.

P 17. Define the function $g(x) = \int_0^x f(t)\, dt$ where the graph of f is shown below.

(a) Determine $g(2)$ and $g(7)$.

(b) Identify the intervals on which $g(x)$ is increasing (support your answer).

(c) Find the critical points of $g(x)$.

P 18. Evaluate

$$\int x^{-2} \cos\left(\frac{\pi}{x}\right) \, dx$$

P 19. Evaluate

$$\int x^2 e^{-x^3} \, dx$$

P 20. Evaluate

$$\int x^4 \ln x \ dx$$

P 21. Find f if $f''(x) = x^{-2}$, $x > 0$, $f(1) = 0$, and $f(2) = 0$.

P 22. Evaluate

$$\int_0^1 \arctan x \ dx.$$

P 23.

(a) If $f(1) = 12$, f' is continuous, and $\int_1^4 f'(x) \ dx = 17$, what is the value of $f(4)$? (b) Find $\dfrac{d}{dx} \left[\displaystyle\int_3^{\sqrt{x}} \dfrac{\cos t}{t} \right].$

P 24. Evaluate

(a) $\displaystyle\int \frac{3x}{(x^2 - 9)^{3/2}}\, dx$

(b) $\displaystyle\int x^2 \sin(x^3) \cos(x^3)\, dx$