3.9 Linear Approximation and the Derivative

Name: Date:

P 1. Find the tangent line approximation for $\sqrt{1+x}$ near $x = 0$.

P 4. Find the local linearization of $f(x) = x^2$ near $x = 1$.

P 6. Show that $1 - x/2$ is the tangent line approximation to $1/\sqrt{1+x}$ near $x = 0$.

P 11.

(a) Find the best linear approximation, $L(x)$, to $f(x) = e^x$ near $x = 0$.

(b) What is the sign of the error, $E(x) = f(x) - L(x)$ for x near 0?

(c) Find the true value of the function at $x = 1$. What is the error? (Give decimal answers.) Illustrate with a graph.

(d) Before doing any calculations, explain which you expect to be larger, $E(0.1)$ or $E(1)$, and why.

(e) Find $E(0.1)$.

P 13.

(a) Graph $f(x) = x^3 - 3x^2 + 3x + 1$.

(b) Find and add to your sketch the local linearization to $f(x)$ at $x = 2$.

(c) Mark on your sketch the true value of $f(1.5)$, the tangent line approximation to $f(1.5)$ and the error in the approximation.

P 17. The equation $x + \ln(1 + x) = 0.2$ has a solution near $x = 0$. By replacing the left side of the equation by its linearization, find an approximate value for the solution.

P 33. Let $f(x) = x^4$ and $a = 1$. Find a formula for the error $E(x)$ in the tangent line approximation to the function near $x = a$. Using a table of values for $E(x)/(x - a)$ near $x = a$, find a value of k such that $E(x)/(x - a) \approx k(x - a)$. Check that, approximately, $k = f''(a)/2$ and that $E(x) = (f''(a)/2)(x - a)^2$.

P 39.

(a) Show that $1 - x$ is the local linearization of $\dfrac{1}{1+x}$ near $x = 0$.

(b) From your answer to part (a), show that near $x = 0$,

$$\frac{1}{1+x^2} \approx 1 - x^2.$$

(c) Without differentiating, what do you think the derivative of $\dfrac{1}{1+x^2}$ is at $x = 0$?