2.6 Differentiability

Name: Date:

P 5. Decide if the functionis differentiable at $x = 0$.

$$f(x) = (x + |x|)^2 + 1$$

P 10. The acceleration due to gravity, g, varies with height above the surface of the earth, in a certain way. If you go down bleow the surface of the earth, g varies in a different way. It can be shown that g is given by

$$g = \begin{cases} \frac{GMr}{R^3}, & \text{for } r < R \\ \frac{GM}{r^2}, & \text{for } r \geq R \end{cases}$$

where R is the radius of the earth, M is the mass of the earth, G is the gravitational constant, and r is the distance to the center of the earth.

(a) Sketch a graph of g against r.

(b) Is g a continuous function of r? Explain your answer.

(c) Is g a differentiable function of r? Explain your answer.

P 14. A cable is made of an insulating material in the shape of a long, thin cylinder of radius r_0. It has electric charge distributed evenly throughtout it. The electric field, E, at a distance r from the center of the cable is given by

$$E = \begin{cases} kr, & \text{for } r \leq r_0 \\ k\frac{r_0^2}{r}, & \text{for } r > r_0. \end{cases}$$

(a) Is E continuous at $r = r_0$?

(b) Is E differentiable at $r = r_0$?

(c) Sketch a graph of E as a function of r.

P 16. Sometimes, odd behavior can be hidden beneath the surface of a rather normal-looking function. Consider the following function:

$$f(x) = \begin{cases} 0, & \text{if } x < 0 \\ x^2, & \text{if } x \geq 0. \end{cases}$$

(a) Sketch a graph of this function. Does it have any vertical segments or corners? Is it differentiable everywhere? If so, sketch the derivative of f' of this function.

(b) Is the derivative function, $f'(x)$, differentiable everywhere? If not, at what point(s) is it not differentiable? Draw the second derivative of $f(x)$ wherever it exists. Is the second derivative function, $f''(x)$, differentiable? Continuous?

P 18. Explain what is wrong with the statement.
"If f is not differentiable at a point then it is not continuous at that point."