3.5 Exponential and Logarithmic Models

Name: Date: June 26, 2013

P 44. POPULATION The table shows the population (in millions) of five countries in 2000 and the projected populations (in millions) for the year 2015. (Source: U.S. Census Bureau)

<table>
<thead>
<tr>
<th>Country</th>
<th>2000</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulgaria</td>
<td>7.8</td>
<td>6.9</td>
</tr>
<tr>
<td>Canada</td>
<td>31.1</td>
<td>35.1</td>
</tr>
<tr>
<td>China</td>
<td>1268.9</td>
<td>1393.4</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>59.5</td>
<td>62.2</td>
</tr>
<tr>
<td>United States</td>
<td>282.2</td>
<td>325.5</td>
</tr>
</tbody>
</table>

(a) Find the exponential growth or decay model for the population of each country by letting \( t = 0 \) correspond to 2000. Use the model to predict the population of each country in 2030.

(b) You can see that the populations of the United States and the United Kingdom are growing at different rates. What constant in the equation \( y = ae^{bt} \) is determined by these different growth rates? Discuss the relationship between the different growth rates and the magnitude of the constant.

(c) You can see that the population of China is increasing while the population of Bulgaria is decreasing. What constant in the equation \( y = ae^{bt} \) reflects this difference? Explain.
P 50. BACTERIA GROWTH The number of bacteria in a culture is increasing according to the law of exponential growth. The initial population is 250 bacteria, and the population after 10 hours is double the population after 1 hour. How many bacteria will there be after 6 hours?

P 52. Carbon 14 dating assumes that the carbon dioxide on Earth today has the same radioactive content as it did centuries ago. If this is true, the amount of $^{14}C$ absorbed by a tree that grew several centuries ago should be the same as the amount of $^{14}C$ absorbed by a tree growing today. A piece of ancient charcoal contains only 15% as much radioactive carbon as a piece of modern charcoal. How long ago was the tree burned to make the ancient charcoal if the half-life of $^{14}C$ is 5715 years?
A cell site is a site where electronic communications equipment is placed in a cellular network for the use of mobile phones. The number $y$ of cell sites from 1985 through 2008 can be modeled by

$$y = \frac{237,101}{1 + 1950e^{-0.355t}}$$

where $t$ represents the year, with $t = 5$ corresponding to 1985. (Source: CTIA-The Wireless Association)

(a) Use the model to find the number of cell sites in the years 1985, 2000, and 2006.

(b) Use a graphing utility to graph the function.

(c) Use the graph to determine the year in which the number of cell sites will reach 235,000.

(d) Confirm your answer to part (c) algebraically.
The Richter scale

\[ R = \log \frac{I}{I_0} \]

is used for measuring the magnitudes of earthquakes. Find the intensity of the earthquake that hit

(a) Southern Sumatra, Indonesia in 2007, measuring 8.5 on the Richter scale (Let \( I_0 = 1 \)).

(b) Illinois in 2008, measuring 5.4 on the Richter scale (Let \( I_0 = 1 \)).

(c) Costa Rica in 2009, measuring 6.1 on the Richter scale (Let \( I_0 = 1 \)).