Homework 5

Name: Due: June 26, 2013

Pledge and Signatures:

P 4.7. 16. Find a general solution to

$$t^2y''(t) - 3ty'(t) + 6y(t) = 0$$

P 4.7. 20. Solve

$$t^2y''(t) + 7ty'(t) + 5y(t) = 0$$

given that $y(1) = -1$ and $y'(1) = 13$.

P 8.3. 14. Find at least the first four nonzero terms in a power series expansion about $x = 0$ for a general solution to

$$(x^2 + 1)y'' + y = 0.$$

P 8.3. 34. Emden's Equation. A classical nonlinear equation that occurs in the study of the thermal behavior of a spherical cloud is **Emden's equation**

$$y'' + \frac{2}{x}y' + y^n = 0$$

with initial conditions $y(0) = 1, y'(0) = 0$. Even though $x = 0$ is not an ordinary point for this equation (which is nonlinear for $n \neq 1$), it turns out that there does exist a solution analytic at $x = 0$. Assuming that n is a positive integer, show that the first few terms of the power series solution are

$$y = 1 - \frac{x^2}{3!} + n\frac{x^4}{5!} + \cdots ,$$

P 10.3. 28.

T 2. *If f and f' are piecewise continuous on $[-L, L]$, then for any x in $(-L, L)$*

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right\} = \frac{1}{2}[f(x^+) + f(x^-)],$$

where

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L}\, dx, \quad n = 0, 1, 2, \ldots$$

and

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L}\, dx, \quad n = 1, 2, 3 \ldots$$

For $x = \pm L$, the series converges to $\frac{1}{2}[f(-L^+) + f(L^-)]$.

(a) Show that the function $f(x) = x^2$ has the Fourier series, on $-\pi < x < \pi$,

$$f(x) \sim \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx.$$

(b) Use the result of part (a) and Theorem 2 to show that

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

(c) Use the result of part (a) and Theorem 2 to show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

P 11.2. 4. Determine a solution, if any, to

$$y'' + 4y' + 8y = 0$$

given that $y'(0) = 0$ and $y'(\pi) = 0$.

P 11.2. 16. Find all real eigenvalues and eigenfunctions for

$$y'' - 2y + \lambda y = 0$$

given that $y(0) = 0$ and $y'(\pi) = 0$.

P 11.2. 24. Solve

$$x^2y'' + \lambda(xy' - y) = 0$$

given that $y(1) = 0$ and $y'(e) = 0$.