Homework 4

Name: Due: June 18, 2013

Pledge and Signatures:

P 4.7. 10. Find a general solution to

$$t^2y''(t) + 7ty'(t) - 7y(t) = 0$$

P 7.6. 36. Solve

$$y'' + 5y' + 6y = tu(t-2)$$

given that $y(0) = 0$ and $y'(0) = 1$.

P 7.7. 22. Solve

$$y'(t) - 2\int_0^t e^{t-v} y(v)\, dv$$

given that $y(0) = 2$.

P 7.8. 20. Solve

$$y'' + 5y' + 6y = e^{-t}\delta(t-2)$$

P 10.2. 14. Find the values of λ for which the given problem has a nontrivial solution. Also determine the corresponding nontrivial solutions.

$$y'' - 2y' + \lambda y = 0$$

given that $0 < x < \pi$, $y(0) = 0$, and $y(\pi) = 0$.

P 10.5. 4. Find a formal solution to

$$\begin{aligned}
&\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0,\\
&\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t) = 0, \quad t > 0,\\
&u(x,0) = x(1-x), \quad 0 < x < 1
\end{aligned}$$

P 10.5. 14. Find a formal solution to

$$\begin{aligned}
&\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2} + 5, \quad 0 < x < \pi, \quad t > 0, \\
&u(0,t) = u(\pi,t) = 1, \quad t > 0, \\
&u(x,0) = 1, \quad 0 < x < \pi
\end{aligned}$$

P 10.6. 4. Find a formal solution to

$$\begin{aligned}
&\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0,\\
&u(0,t) = u(\pi,t) = 0, \quad t > 0,\\
&u(x,0) = \sin 4x + 7\sin 5x, \quad 0 < x < \pi,\\
&\frac{\partial u}{\partial t}(x,0) = \begin{cases} x, & 0 < x < \pi/2,\\ \pi - x, & \pi/2 < x < \pi \end{cases}
\end{aligned}$$

P 10.6. 8. Find a formal solution to

$$\begin{aligned}
&\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + x \sin t, \quad 0 < x < \pi, \quad t > 0, \\
&u(0,t) = u(\pi,t) = 0, \quad t > 0, \\
&u(x,0) = 0, \quad 0 < x < \pi, \\
&\frac{\partial u}{\partial t}(x,0) = 0, 0 < x < \pi
\end{aligned}$$

P 10.7. 4. Find a formal solution to

$$\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < \pi, \quad 0 < y < \pi, \\
&u(0, y) = u(\pi, y) = 0, \quad 0 \le y \le \pi, \\
&u(x, 0) = \sin x + \sin 4x, \quad 0 \le x \le \pi, \\
&u(x, \pi) = 0, \quad 0 \le x \le \pi
\end{aligned}$$

P 10.7. 12. Find a formal solution to

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0, \quad 1 < r, \quad -\pi \leq \theta \leq \pi,$$
$$u(1, \theta) = f(\theta), \quad -\pi \leq \theta \leq \pi,$$
$$u(r, \theta) \text{ remains bounded as } r \to \infty$$