# Homework 3

*Pledge and Signatures*:

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**P 4.1.   8.** Find a synchronous solution of the form $A\cos\Omega t + B\sin\Omega t$ to the given forced oscillator equation using the methods of Example 4 to solve for $A$ and $B$.

$$y'' + 2y' + 5y = -50\sin 5t, \quad \Omega = 5.$$

**P 4.2. 2.** Find a general solution to

$$2y'' + 7y' - 4y = 0.$$

**P 4.2. 14.** Solve

$$y'' + y' = 0$$

given that $y(0) = 2$ and $y'(0) = 1$.

**P 4.3. 10.** Find a general solution to

$$y'' - 8y' + 7y = 0.$$

**P 4.3. 28.** To see the effect of changing the parameter $b$ in the initial value problem

$$y'' + by' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 0,$$

Solve the problem for $b = 5, 4$, and $2$ and sketch the solutions.

**P 4.3. 32. Vibrating Spring without Damping.** A vibrating spring without damping can be modeled by the initial value problem

$$my''(t) + by'(t) + ky(t) = 0,$$

by taking $b = 0$.

(a) If $m = 10$ kg, $k = 250$kg/sec$^2$, $y(0) = 0.3$ m, and $y'(0) = -0.1$ m/sec, find the equation of motion for this undamped vibrating spring.

(b) When the equation of motion is of the form

$$y(t) = c_1 e^{\alpha t} \cos \beta t + c_2 e^{\alpha t} \sin \beta t,$$

the motion is said to be **oscillatory** with **frequency** $\beta/2\pi$. Find the frequency of oscillation for the spring system of part (a).

**P 4.4. 10.** Find a particular solution to

$$y'' + 3y = -9.$$

**P 4.4. 14.** Find a particular solution to

$$2z'' + z = 9e^{2t}.$$

**P 4.5. 18.** Find a general solution to

$$y'' - 2y' - 3y = 3t^2 - 5.$$

**P 4.5. 22.** Find a general solution to

$$y''(x) + 6y'(x) + 10y(x) = 10x^4 + 24x^3 + 2x^2 - 12x + 18.$$

**P 4.5. 30.** Solve
$$y'' + 2y' + y = t^2 + 1 - e^t,$$
given that $y(0) = 0$ and $y'(0) = 2$.

**P 4.6. 2.** Find a general solution to

$$y'' + 4y = \tan 2t$$

**P 4.6. 6.** Find a general solution to

$$y''(\theta) + 16y(\theta) = \sec 4\theta.$$

**P 6.1.** **16.** Verify that the given functions form a fundamental solution set for the given differential equation.

$$y''' - y'' + 4y' - 4y = 0; \quad \{e^x, \cos 2x, \sin 2x\}$$

**P 6.1. 20.** A particular solution and a fundamental solution are given for a nonhomogeneous equation and its corresponding homogeneous equation. (a) Find a general solution to the non-homogeneous equation. (b) Fidn the solution that satisfies the specified initial conditions.

$$xy''' - y'' = -2; \ y(1) = 2, \ y'(1) = -1, \ y''(1) = -4; \ y_p = x^2; \ \{1, x, x^3\}$$

**P 6.2. 4.** Find a general solution for

$$y''' + 2y'' - 19y' - 20y = 0$$

with $x$ as the independent variable.

**P 6.2. 6.** Find a general solution for

$$y''' - y'' + 2y = 0$$

with $x$ as the independent variable.

**P 6.2. 20.** Solve

$$y''' + 7y'' + 14y' + 8y = 0,$$

given that $y(0) = 1$, $y'(0) = -3$, and $y''(0) = 13$.