Homework 2

Name: Due: May 28, 2013

Pledge and Signatures:

P 2.2. 8. Solve

$$x\frac{dy}{dx} = \frac{1}{y^3}$$

P 2.2. 16. Solve

$$(x + xy^2)\, dx + e^{x^2} y\, dy = 0$$

P 2.2. 18. Solve

$$\frac{dy}{dx} = (1 + y^2) \tan x, \quad y(0) = \sqrt{3}$$

P 2.2. 38. Free Fall. A model for an object falling toward Earth is described by

$$m\frac{dv}{dt} = mg - bv,$$

assuming that only air resistance and gravity are acting on the object and v, m, g, and $b > 0$ are the velocity, mass, gravity, and some constant, respectively. If $m = 100$kg, $g = 9.8$ m/sec^2, $b = 5$ kg/sec, and $v(0) = 10$ m/sec, solve for $v(t)$. What is the limiting (i.e. terminal) velocity of the object?

P 2.3. 8. Find the general solution of

$$\frac{dy}{dx} - y - e^{3x} = 0$$

P 2.3. 14. Find the general solution of

$$x\frac{dy}{dx} + 3(y + x^2) = \frac{\sin x}{x}$$

P 2.3. 18. Solve

$$\frac{dy}{dx} + 4y - e^{-x} = 0, \quad y(0) = \frac{4}{3}$$

P 2.3. 20. Solve

$$\frac{dy}{dx} + \frac{3y}{x} + 2 = 3x, \quad y(1) = 1$$

P 2.3. 22. Solve

$$\sin x \frac{dy}{dx} + y \cos x = x \sin x, \quad y\left(\frac{\pi}{2}\right) = 2$$

P 2.3. 30. Bernoulli Equations. The equation

$$\frac{dy}{dx} + 2y = xy^{-2}$$

is an example of a Bernoulli equation.

(a) Show that the substitution $v = y^3$ reduces the given equation to

$$\frac{dv}{dx} + 6v = 3x.$$

(b) Solve the resulting equation for v. Then make the substitution $v = y^3$ to obtain the solution to the given equation.

P 2.4. 10. Determine whether

$$(2xy + 3)\,dx + (x^2 - 1)\,dy = 0$$

is exact. If it is, then solve it.

P 2.4. 14. Determine whether

$$e^t(y - t)\,dt + (1 + e^t)\,dy = 0$$

is exact. If it is, then solve it.

P 2.4. 22. Solve

$$(ye^{xy} - 1/y)\,dx + (xe^{xy} + x/y^2)\,dy = 0, \quad y(1) = 1$$

P 2.4. 28. For each of the following equations, find the most general function $N(x, y)$ so that the equation is exact.

(a) $[y\cos(xy) + e^x]\, dx + N(x, y)\, dy = 0$

(b) $(ye^{xy} - 4x^3y + 2)\, dx + N(x, y)\, dy = 0$

P 2.4. 30. Consider the equation

$$(5x^2y + 6x^3y^2 + 4xy^2)\,dx + (2x^3 + 3x^4y + 3x^2y)\,dy = 0.$$

(a) Show that the equation is not exact.

(b) Multiply the equation by $x^n y^m$ and determine values of n and m that make the resulting equation exact.

(c) Use the solution of the resulting exact equation to solve the original equation.

P 2.5. 10. Solve

$$(2y^2 + 2y + 4x^2)\, dx + (2xy + x)\, dy = 0$$

P 2.5. 14. Solve

$$(12 + 5xy)\, dx + (6xy^{-1} + 3x^2)\, dy = 0$$

P 2.6. 24. Solve

$$\frac{dy}{dx} + \frac{y}{x-2} = 5(x-2)y^{1/2}$$

P 2.6. 26. Solve

$$\frac{dy}{dx} + y = e^x y^{-2}$$

P 2.6. 28. Solve

$$\frac{dy}{dx} + y^3 x + y = 0$$

P 2.6. 32. Solve

$$(2x + y + 4)\, dx + (x - 2y - 2)\, dy = 0$$

P 2.6. 36. Solve

$$(t + x + 2)\, dx + (3t - x - 6)\, dt = 0$$

P 2.6. 40. Solve

$$(y^3 - \theta y^2)\, d\theta + 2\theta^2 y\, dy = 0$$

RP 4. Solve

$$\frac{dy}{dx} + \frac{3y}{x} = x^2 - 4x + 3$$

RP 8. Solve

$$\frac{dy}{dx} + \frac{2y}{x} = 2x^2y^2$$

RP 12. Solve

$$(y^3 + 4e^x y)\,dx + (2e^x + 3y^2)\,dy = 0$$

RP 16. Solve

$$\frac{dy}{dx} + y \tan x + \sin x = 0$$

RP 20. Solve

$$(x^2 - 3y^2)\, dx + 2xy\, dy = 0$$

RP 24. Solve

$$(\sqrt{y/x} + \cos x)\, dx + (\sqrt{x/y} + \sin y)\, dy = 0$$

RP 28. Solve

$$\frac{dy}{dx} = \frac{x - y - 1}{x + y + 5}$$

RP 32. Solve

$$\frac{dy}{dx} = \left(\frac{x}{y} + \frac{y}{x}\right), \quad y(1) = -4$$

RP 36. Solve

$$[2\cos(2x+y) - x^2]\,dx + [\cos(2x+y) + e^y]\,dy = 0, \quad y(1) = 0$$

RP 40. Solve

$$\frac{dy}{dx} - 4y = 2xy^2, \quad y(0) = -4$$