Homework 1

Name: Due: May 22, 2013

Pledge and Signatures:

P 1.2. 1.

(a) Show that $y^2 + x - 3 = 0$ is an implicit solution to $dy/dx = -1/(2y)$ on the interval $(-\infty, 3)$.

(b) Show that $xy^3 - xy^3 \sin x = 1$ is an implicit solution to

$$\frac{dy}{dx} = \frac{(x \cos x + \sin x - 1)y}{3(x - x \sin x)}$$

P 1.2. 10. Determine whether $x^2 + y^2 = 4$ is an implicit solution to

$$\frac{dy}{dx} = \frac{x}{y}$$

P 1.2. 20. Determine for which values of m the function $\phi(x) = e^{mx}$ is a solution to the given equation.

(a) $\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 5y = 0$

(b) $\dfrac{d^3y}{dx^3} + 3\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} = 0$

P 1.2. 5. The logistic equation for the population (in thousands) of a certain species is given by

$$\frac{dp}{dt} = 3p - 2p^2.$$

(a) Sketch the direction field by using either a computer software package or the method of isoclines.

(b) If the initial population is 3000 [That is, p(0) = 3], what can you say about the limiting population $\lim_{t\to\infty} p(t)$?

(c) If $p(0) = 0.8$, what is $\lim_{t\to\infty} p(t)$?

(d) Can a population of 2000 ever decline to 800?

P 1.3. 6. Use Euler's Method with step size $h = 0.2$ to approximate the solution to the initial value problem

$$y' = \frac{1}{x}(y^2 + y), \quad y(1) = 1$$

at the points $x = 1.2, 1.4, 1.6$, and 1.8.

CP 1. Using Taylor series method, solve the equation $y' + 3y = x^2$, $y(0) = 1$.

CP 2. Sketch the phase portrait for the equation $y' = y(1 + y)(2 - y)$.