2.4 Exact Equations

Name: Date: May 28, 2013

P 10. Determine whether
$$(2xy + 3)\, dx + (x^2 - 1)\, dy = 0$$
is exact. If it is, then solve it.

P 11. Determine whether
$$(\cos x \cos y + 2x)\, dx - (\sin x \sin y + 2y)\, dy = 0$$
is exact. If it is, then solve it.

P 13. Determine whether
$$(t/y)\, dy + (1 + \ln y)\, dt = 0$$
is exact. If it is, then solve it.

P 14. Determine whether
$$e^t(y - t)\, dt + (1 + e^t)\, dy = 0$$
is exact. If it is, then solve it.

P 19. Determine whether

$$\left(2x + \frac{y}{1 + x^2y^2}\right) dx + \left(\frac{x}{1 + x^2y^2 - 2y}\right) dy = 0$$

is exact. If it is, then solve it.

P 22. Solve

$$(ye^{xy} - 1/y) \, dx + (xe^{xy} + x/y^2) \, dy = 0, \quad y(1) = 1$$

P 23. Solve

$$(e^t y + te^t y)\, dt + (te^t + 2)\, dy = 0, \quad y(0) = -1$$

P 25. Solve

$$(y^2 \sin x) \, dx + (1/x - y/x) \, dy = 0, \quad y(\pi) = 1$$

P 27. For each of the following equations, find the most general function $M(x, y)$ so that the equation is exact.

(a) $M(x, y) \, dx + (\sec^2 y - x/y) \, dy = 0$

(b) $M(x, y) \, dx + (\sin x \cos y - xy - e^{-y}) \, dy = 0$

P 28. For each of the following equations, find the most general function $N(x, y)$ so that the equation is exact.

(a) $[y \cos(xy) + e^x] \, dx + N(x, y) \, dy = 0$

(b) $(ye^{xy} - 4x^3y + 2) \, dx + N(x, y) \, dy = 0$

P 29. Consider the equation
$$(y^2 + 2xy)\, dx - x^2\, dy = 0$$

(a) Show that this equation is not exact.

(b) Show that multiplying both sides of the equation by y^{-2} yields a new equation that is exact.

(c) Use the solution of the resulting exact equation to solve the original equation.

(d) Were any solutions lost in the process?

P 30. Consider the equation

$$(5x^2y + 6x^3y^2 + 4xy^2)\, dx + (2x^3 + 3x^4y + 3x^2y)\, dy = 0.$$

(a) Show that the equation is not exact.

(b) Multiply the equation by $x^n y^m$ and determine values of n and m that make the resulting equation exact.

(c) Use the solution of the resulting exact equation to solve the original equation.