2.3 Linear Equations

Name: Date: May 28, 2013

P 7. Find the general solution of

$$\frac{dy}{dx} = \frac{y}{x} + 2x + 1$$

P 8. Find the general solution of

$$\frac{dy}{dx} - y - e^{3x} = 0$$

P 14. Find the general solution of

$$x\frac{dy}{dx} + 3(y + x^2) = \frac{\sin x}{x}$$

P 15. Find the general solution of

$$(x^2 + 1)\frac{dy}{dx} + xy - x = 0$$

P 18. Solve

$$\frac{dy}{dx} + 4y - e^{-x} = 0, \quad y(0) = \frac{4}{3}$$

P 20. Solve

$$\frac{dy}{dx} + \frac{3y}{x} + 2 = 3x, \quad y(1) = 1$$

P 21. Solve

$$\cos x \frac{dy}{dx} + y \sin x = 2x \cos^2 x, \quad y\left(\frac{\pi}{4}\right) = \frac{-15\sqrt{2}\pi^2}{32}$$

P 22. Solve

$$\sin x \frac{dy}{dx} + y \cos x = x \sin x, \quad y\left(\frac{\pi}{2}\right) = 2$$

P 30. Bernoulli Equations. The equation

$$\frac{dy}{dx} + 2y = xy^{-2}$$

is an example of a Bernoulli equation.

(a) Show that the substitution $v = y^3$ reduces the given equation to

$$\frac{dv}{dx} + 6v = 3x.$$

(b) Solve the resulting equation for v. Then make the substitution $v = y^3$ to obtain the solution to the given equation.

P 33. Singular Points. Those values of x for which $p(x)$ in

$$\frac{dy}{dx} + P(x)y = Q(x)$$

is not defined are called **singular points** of the equation. For example. $x = 0$ is a singular point of the equation $xy' + 2y = 3x$, since when the equation is written in the standard form, $y' + (2/x)y = 3$, we see that $P(x) = 2/x$ is not defined at $x = 0$. On an interval containing a singular point, the questions of the existence and uniqueness of a solution are left unanswered, sine the existence and uniqueness theorem does not apply. To show the possible behavior of solutions near a singular point, consider the following equations.

(a) Show that $xy' + 2y = 3x$ has only one solution defined at $x = 0$. Then show that the initial value problem for this equation with initial condition $y(0) = y_0$ has a unique solution when $y_0 = 0$ and no solution when $y_0 \neq 0$.

(b) Show that $xy' - 2y = 3x$ has an infinite number of solutions defined at $x = 0$. Then show that the initial value problem for this equation with initial conditions $y(0) = 0$ has an infinite number of solutions.