1.4 The Approximation Method of Euler

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P 6. Use Euler's Method with step size $h = 0.2$ to approximate the solution to the initial value problem

$$y' = \frac{1}{x}(y^2 + y), \quad y(1) = 1$$

at the points $x = 1.2, 1.4, 1.6$, and 1.8.

P 10. Find a value of h for Euler's method such that $y(1)$ is approximated to within ± 0.01, if $y(x)$ satisfies the initial value problem

$$y' = x - y, \quad y(0) = 0.$$

Also, find to within ± 0.05, the value of x_0 such that $y(x_0) = 0.2$. Compare your answers with those given by the actual solution $y - e^{-x} + x - 1$. Graph the polygonal-line approximation and the actual solution on the same coordinate system.

P 15. Newton's Law of Cooling. Newton's law of cooling states that the rate of change of the temperature $T(t)$ of a body is proportional to the difference between the temperature of the medium $M(t)$ and the temperature of the body. That is,

$$\frac{dT}{dt} = K[M(t) - T(t)],$$

where K is a constant. Let $k = 1(min)^{-1}$ and the temperature of the medium be constant, $M(t) \equiv 70°$. If the body is initially $100°$, use Euler's method with $h = 0.1$ to approximate the temperature of the body after

(a) 1 minute.

(b) 2 minutes.

P 16. Stefan's Law of Radiation. Stefan's law of radiation states that the rate of change in temperature of a body at $T(t)$ degrees in a medium at $M(t)$ degrees is proportional to $M^4 - T^4$. That is,

$$\frac{dT}{dt} = K(M(t)^4 - T(t)^4),$$

where K is a constant. Let $K = (40)^{-4}$ and assume that the medium temperature is constant, $M(t) \equiv 70°$. If $T(0) = 100°$, use Euler's method with $h = 0.1$ to approximate $T(1)$ and $T(2)$.