P 3. A model for the velocity $v$ at time $t$ of a certain object falling under the influence of gravity in a viscous medium is given by the equation

$$\frac{dv}{dt} = 1 - \frac{v}{8}.$$ 

From the direction field shown below, sketch the solutions with the initial conditions $v(0) = 5, 8,$ and 15. Why is the value $v = 8$ called the “terminal velocity”?

Figure 1: Direction field for $\frac{dv}{dt} = 1 - \frac{v}{8}$
P 4. If the viscous force in P3 is nonlinear, a possible model would be provided by the differential equation

$$\frac{dv}{dt} = 1 - \frac{v^3}{8}.$$ 

Redraw the direction field in P3 to incorporate this $v^3$ dependence. Sketch the solutions with initial conditions $v(0) = 0, 1, 2, 3$. What is the terminal velocity in this case?
The logistic equation for the population (in thousands) of a certain species is given by
\[ \frac{dp}{dt} = 3p - 2p^2. \]

(a) Sketch the direction field by using either a computer software package or the method of isoclines.

(b) If the initial population is 3000 [That is, \( p(0) = 3 \)], what can you say about the limiting population \( \lim_{t \to \infty} p(t) \)?

(c) If \( p(0) = 0.8 \), what is \( \lim_{t \to \infty} p(t) \)?

(d) Can a population of 2000 ever decline to 800?
Consider the differential equation
\[ \frac{dp}{dt} = p(p - 1)(p - 2) \]
for the population \( p \) (in thousands) of a certain species at time \( t \).

(a) Sketch the direction field by using either a computer software package or the method of isoclines.

(b) If the initial population is 4000 [that is, \( p(0) = 4 \)], what can you say about the limiting population \( \lim_{t \to \infty} p(t) \)?

(c) If \( p(0) = 1.7 \), what is \( \lim_{t \to \infty} p(t) \)?

(d) If \( p(0) = 0.8 \), what is \( \lim_{t \to \infty} p(t) \)?

(e) Can a population of 900 ever increase to 1100?
P 9. Let \( \phi(x) \) denote the solution to the initial value problem

\[
\frac{dy}{dx} = x - y, \quad y(0) = 1.
\]

(a) Show that \( \phi''(x) = 1 - \phi'(x) = 1 - x + \phi(x) \).

(b) Argue that the graph of \( \phi \) is decreasing for \( x \) near zero and that as \( x \) increases from zero, \( \phi(x) \) decreases until it crosses the line \( y = x \), where its derivative is zero.

(c) Let \( x^* \) be the abscissa of the point where the solution curve \( y = \phi(x) \) crosses the line \( y = x \). Consider the sign of \( \phi''(x^*) \) and argue that \( \phi \) has a relative minimum at \( x^* \).

(d) What can you say about the graph of \( y = \phi(x) \) for \( x > x^* \)?

(e) Verify that \( y = x - 1 \) is a solution to \( \frac{dy}{dx} = x - y \) and explain why the graph of \( \phi(x) \) always stays above the line \( y = x - 1 \).

(f) Sketch the direction field for \( \frac{dy}{dx} = x - y \) by using the method of isoclines or a computer software package.

(g) Sketch the solution \( y = \phi(x) \) using the direction field in (f).
P 17. From a sketch of the direction field, what can one say about the behavior as \( x \) approaches \( \infty \) of a solution the the following?

\[
\frac{dy}{dx} = 3 - y + \frac{1}{x}
\]

P 18. From a sketch of the direction field, what can one say about the behavior as \( x \) approaches \( \infty \) of a solution the the following?

\[
\frac{dy}{dx} = -y
\]