1.3 Direction Fields

Name: Date: May 21, 2013

P 3. A model for the velocity v at time t of a certain object falling under the influence of gravity in a viscous medium is given by the equation

$$\frac{dv}{dt} = 1 - \frac{v}{8}.$$

From the direction field shown below, sketch the solutions with the initial conditions $v(0) = 5, 8,$ and 15. Why is the value $v = 8$ called the "terminal velocity"?

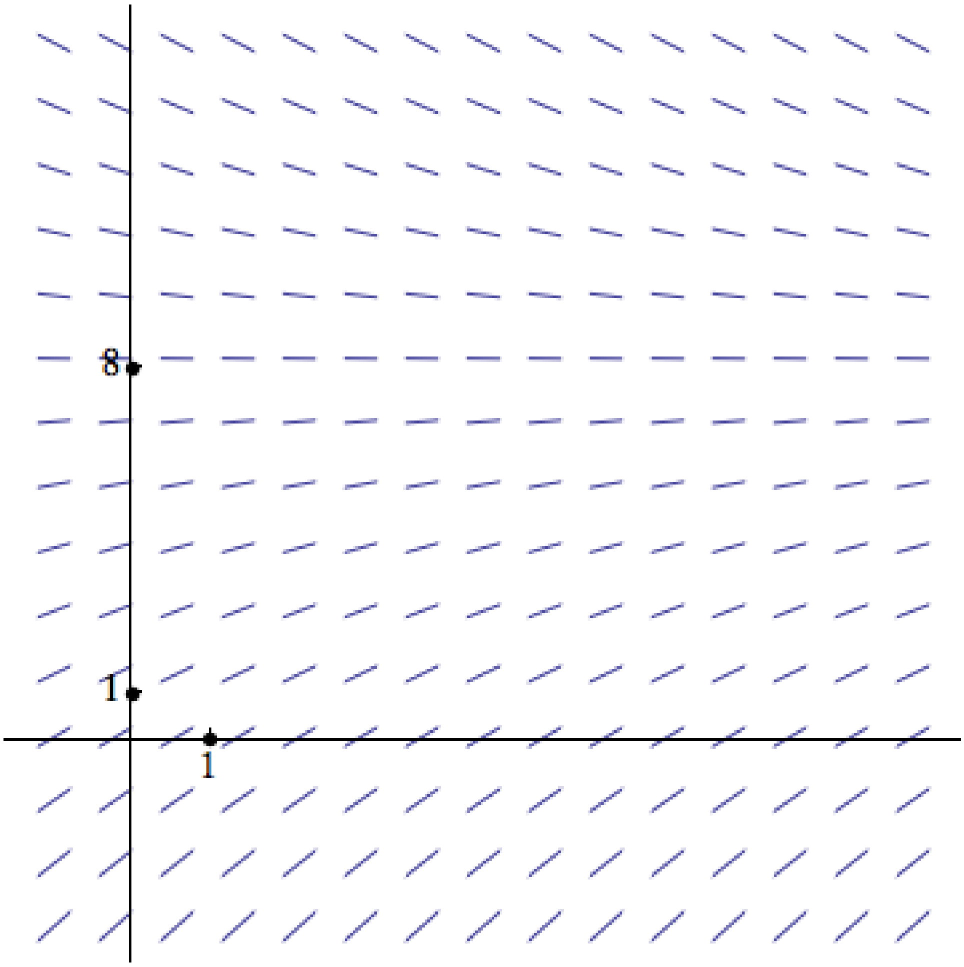

Figure 1: Direction field for $\frac{dv}{dt} = 1 - \frac{v}{8}$

P 4. If the viscous force in **P3** is nonlinear, a possible model would be provided by the differential equation

$$\frac{dv}{dt} = 1 - \frac{v^3}{8}.$$

Redraw the direction field in **P3** to incorporate this v^3 dependence. Sketch the solutions with initial conditions $v(0) = 0, 1, 2, 3$. What is the terminal velocity in this case?

P 5. The logistic equation for the population (in thousands) of a certain species is given by

$$\frac{dp}{dt} = 3p - 2p^2.$$

(a) Sketch the direction field by using either a computer software package or the method of isoclines.

(b) If the initial population is 3000 [That is, p(0) = 3], what can you say about the limiting population $\lim_{t\to\infty} p(t)$?

(c) If $p(0) = 0.8$, what is $\lim_{t\to\infty} p(t)$?

(d) Can a population of 2000 ever decline to 800?

P 7. Consider the differential equation

$$\frac{dp}{dt} = p(p-1)(p-2)$$

for the population p (in thousands) of a certain species at time t.

(a) Sketch the direction field by using either a computer software package or the method of isoclines.

(b) If the initial population is 4000 [that is, $p(0) = 4$], what can you say about the limiting population $\lim_{t\to\infty} p(t)$?

(c) If $p(0) = 1.7$, what is $\lim_{t\to\infty} p(t)$?

(d) If $p(0) = 0.8$, what is $\lim_{t\to\infty} p(t)$?

(e) Can a population of 900 ever increase to 1100?

P 9. Let $\phi(x)$ denote the solution to the initial value problem

$$\frac{dy}{dx} = x - y, \quad y(0) = 1.$$

(a) Show that $\phi''(x) = 1 - \phi'(x) = 1 - x + \phi(x)$.

(b) Argue that the graph of ϕ is decreasing for x near zero and that as x increases from zero, $\phi(x)$ decreases until it crosses the line $y = x$, where its derivative is zero.

(c) Let x^* be the abscissa of the point where the solution curve $y = \phi(x)$ crosses the line $y = x$. Consider the sign of $\phi''(x^*)$ and argues that ϕ has a relative minimum at x^*.

(d) What can you say about the graph of $y = \phi(x)$ for $x > x^*$?

(e) Verifty that $y = x - 1$ is a solution to $dy/dx = x - y$ and explain why the graph of $\phi(x)$ always stays above the line $y = x - 1$.

(f) Sketch the direction field for $dy/dx = x - y$ by using the method of isoclines or a computer software package.

(g) Sketch the solution $y = \phi(x)$ using the direction field in (f).

P 17. From a sketch of the direction field, what can one say about the behavior as x approaches ∞ of a solution the the following?

$$\frac{dy}{dx} = 3 - y + \frac{1}{x}$$

P 18. From a sketch of the direction field, what can one say about the behavior as x approaches ∞ of a solution the the following?

$$\frac{dy}{dx} = -y$$