1.2 Solutions and Initial Value Problems

Name: Date: May 21, 2013

P 1.

(a) Show that $y^2 + x - 3 = 0$ is an implicit solution to $dy/dx = -1/(2y)$ on the interval $(-\infty, 3)$.

(b) Show that $xy^3 - xy^3 \sin x = 1$ is an implicit solution to

$$\frac{dy}{dx} = \frac{(x \cos x + \sin x - 1)y}{3(x - x \sin x)}$$

P 2.

(a) Show that $\phi(x) = x^2$ is an explicit solution to

$$x\frac{dy}{dx} = 2y$$

on the interval $(-\infty, \infty)$.

(b) Show that $\phi(x) = e^x - x$ is an explicit solution to

$$\frac{dy}{dx} + y^2 = e^{2x} + (1 - 2x)e^x + x^2 - 1$$

on the interval $(-\infty, \infty)$.

P 9. Determine whether $y - \ln y = x^2 + 1$ is an implicit solution to

$$\frac{dy}{dx} = \frac{2xy}{y-1}$$

P 10. Determine whether $x^2 + y^2 = 4$ is an implicit solution to

$$\frac{dy}{dx} = \frac{x}{y}$$

P 20. Determine for which values of m the function $\phi(x) = e^{mx}$ is a solution to the given equation.

(a) $\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 5y = 0$

(b) $\dfrac{d^3y}{dx^3} + 3\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} = 0$

P 21. Determine for which values of m the function $\phi(x) = x^m$ is a solution to the given equation.

(a) $3x^2 \dfrac{d^2y}{dx^2} + 11x\dfrac{dy}{dx} - 3y = 0$

(b) $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 5y = 0$

P 23. Determine whether the Existence and Uniqueness theorem implies that the initial value problem below has a unique solution.

$$\frac{dy}{dx} = y^4 - x^4, \quad y(0) = 7$$

P 24. Determine whether the Existence and Uniqueness theorem implies that the initial value problem below has a unique solution.

$$\frac{dy}{dt} - ty = \sin^2 t, \quad y(\pi) = 5$$

P 25. Determine whether the Existence and Uniqueness theorem implies that the initial value problem below has a unique solution.

$$3x\frac{dx}{dt} + 4t = 0, \quad x(2) = -\pi$$

P 26. Determine whether the Existence and Uniqueness theorem implies that the initial value problem below has a unique solution.

$$\frac{dx}{dt} + \cos x = \sin t, \quad x(\pi) = 0$$

P 30. Implicit Function Theorem. Let $G(x, y)$ have continuous first partial derivatives in the rectangle $R = \{(x, y) \mid a < x < b,\ c < y < d\}$ containing the point (x_0, y_0). If $G(x_0, y_0) = 0$ and the partial derivatives $G_y(x_0, y_0) \neq 0$, then there exists a differentiable function $y = \phi(x)$, defined in some interval $I = (x_0 - \delta, x_0 + \delta)$, that satisfies $G(x, \phi(x)) = 0$ for all $x \in I$.

The implicit function theorem gives conditions under which the relationship $G(x, y) = 0$ defines y implicitly as a function of x. Use the implicit function theorem to show that the relationship $x + y + e^{xy} = 0$, defines y implicitly as a function of x near $(0, -1)$.

P 31. Consider

$$y\frac{dy}{dx} - 4x = 0. \tag{1}$$

(a) Does the Existence and Uniqueness theorem imply the existence of a unique solution to (1) that satisfies $y(x_0) = 0$?

(b) Show that when $x_0 \neq 0$, equation (1) can't possibly have a solution in a neighborhood of $x = x_0$ that satisfies $y(x_0) = 0$.

(c) Show that there are two distinct solutions to (1) satisfying $y(0) = 0$.