Final Exam Review Problems

P 1. Find the critical points of $f(x, y) = x^2y + 2y^2 - 8xy + 11$ and classify them.

P 2. Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $x + z = 3$.

P 3. Consider the integral

$$\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy.$$

(a) Sketch the domain of integration.

(b) Write the equivalent integral by reversing the order of integration.

(c) Evaluate the integral.

P 4. Find the absolute maximum and minimum values of $f(x, y) = x^2 + xy + y^2$ on the region $\{(x, y) \mid x^2 + y^2 \leq 1\}$.

P 5. Find the directional derivative of the function $f(x, y, z) = x^2 + x\sqrt{1+z}$ at $(1, 2, 3)$ in the direction $\vec{v} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$

P 6. Find the maximum and minimum value of $f(x, y) = x^2 y$ subject to the constraint $x^2 + y^2 = 1$.

P 7. Consider the surface defined by the equation

$$xz + y^2 + xyz - z^2 = 0.$$

(a) Determine an equation for the tangent plane at the point $P(1, -1, -1)$.

(b) Use the tangent plane to estimate the value of z when $x = 1.05$ and $y = -1.10$.

P 8. For the function $f(x, y) = e^{-x^2-y^2}$

(a) Determine the volume under the graph $z = f(x, y)$ and above the disc $x^2 + y^2 \leq a^2$.

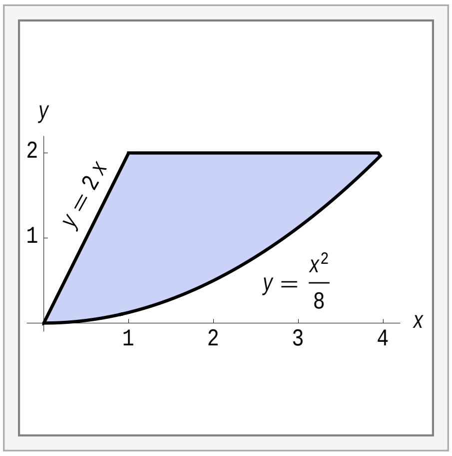
(b) What happens to this volume as $a \to \infty$?

P 9. Consider the function $z = f(x, y) = xy^2 - x$. Find the maximum and minimum values of f on the closed disc $x^2 + y^2 \leq 4$.

P 10. Evaluate the double integral

$$\iint_D 4xy^2 \, dA$$

where D is the shaded region in the figure.



P 11. Let p be the joint density function such that $p(x, y) = xy$ in R, the rectangle $0 \le x \le 1$, $0 \le y \le 2$, and $p(x, y) = 0$ outside R. Find the fraction of the population satisfying the constraint $0 \le x \le 1/3$ and $0 \le y \le 1/3$.

P 12.

(a) Find a contsant k such that $f(x,y) = k(x+y)$ is a probability density function on the quarter disk $x^2 + y^2 \leq 121$, $x \geq y$, and $y \geq 0$.

(b) Find the probability that a point chosen in the quarter disk according to the density in part (a) is less than 5 units from the origin.

P 13.

(a) Let $f(x,y) = 9\cos x \sin y$ and let S be the surface $z = f(x,y)$. Find a unit vector $\vec{u}$ that is normal to the surface S at the point $(0, \pi/2, 9)$.

(b) What is an equation of the tangent lane to the surface S at the point $(0, \pi/2, 9)$?

P 14. Find the differential of $g(u, v) = u^2 e^u + uv \sin(u + v)$.

P 15. The concentration of salt in a fluid at (x, y, z) is given by $F(x, y, z) = x^5 + y^{11} + x^5 z^2$ mg/cm^3. You are at the point $(-1, 1, 1)$.

(a) In which direction should you move if you want the concentration to increase the fastest?

(b) You start to move in this direction at a speed of 4 cm/sec. How fast is the concentration changing?

P 16. Evaluate $\int_R \sin(x^2 + y^2)\, dA$ where R is the disk of radius 8 centered at the origin.

P 17. Find the volume under the paraboloid $z = x^2 + y^2$ and above the region R in the plane $z = 0$, where $R = \{(x, y) \mid -2 \leq x \leq 2,\ 0 \leq y \leq 1\}$.

P 18. For the function $F(x,y,z) = x^2y + 2x(1+z)$, determine the following:

(a) the gradient of F at the point $P(1,-1,3)$;

(b) the rate of change of F in the direction of vector $\vec{a} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$;

(c) the equation of the tangent plane to the level durface $F(x,y,z) = 7$, at the point $P(1,-1,3)$.

P 19.

(a) Verify that $u(t,x) = \cos(2x - t + 3)$ is a solution to the equation, $2u_t + u_x = 0$.

(b) Suppose $z = xy + f(u(x,y))$, where f is a differentiable function of u. If $u(x,y) = x^2 + y$, show that z satisfies the differential equation,

$$\frac{\partial z}{\partial x} - 2x\frac{\partial z}{\partial y} = y - 2x^2$$

P 20. Find the absolute maximum and minimum values of $f(x,y) = 3 + xy - x - 2y$ on the triangular region with vertices $(0,0), (4,0)$, and $(2,3)$.

P 21. Evaluate the double interal $\iint_D xy \; dA$ where D is the triangular region with vertices $(0, 0), (2, 1)$, and $(3, 0)$.

P 22. Consider the surface defined as the level set $x^3 + y^3 + z^3 + 4xyz = 0$, and $P(1, -1, 2)$ a point on this surface. Assume that close to P the surface determines an implicit function $z = h(x, y)$.

(a) Determine the gradient of h at P.

(b) Determine a direction P (a direction with respect to x and y) such that the rate of change of h is zero.

P 23. A closed rectangular box has volume 40 cm^3. What are the lengths of the edges giving the minimum surface area?

P 24. Find the maximum and minimum values of $f(x, y, z) = x^2 - 18y + 20z^2$ subject to the constraint $x^2+y^2+z^2 = 1$, if such values exist. If there is no global maximum or global minimum, state so.

P 25. The values of $f(x, y)$ are in the table below.

y \ x	4.0	4.1	4.2
1.0	6	8	10
1.2	5	7	9
1.4	4	6	5

Let $R = \{(x, y) \mid 4 \le x \le 4.2 \text{ and } 1 \le y \le 1.4\}$. Find a reasonable over and under-estimates for

$$\int_R f(x, y)\, dA.$$