Exam 1 Review Problems

Name: Date:

P 1. Find $\frac{\partial R}{\partial x}$ and $\frac{\partial R}{\partial y}$ for

$$R = \ln\left(u^2 + v^2 + w^2\right),$$

when $x = y = 1$ and where $u = x + 2y$, $v = 2x - y$, and $w = 2xy$.

P 2. Consider the function $u = \dfrac{1}{x + at}$, defined when $x + at \neq 0$.

(a) Determine whether or not u satisfies the wave equation,

$$u_{tt} = a^2 u_{xx}.$$

(b) Show that $u_{tx} = u_{xt}$.

P 3. Let $f(x, y) = x^2y + x \ln y$, with $x = s + 2t$ and $y = 3st$. Find $\dfrac{\partial f}{\partial t}$.

P 4. Consider the equation $2x^2yz = 3xy + xz - yz$. Assume that it determines a function $z = f(x, y)$. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

P 5. Consider the function

$$f(x, y, z) = e^{xy} \sin(x - z).$$

(a) Find the rate of change of f at the point $(2, 1, 2)$ in the direction of the vector $\langle 2, 1, 3 \rangle$.

(b) What is the maximum rate of change at $(2, 1, 2)$? In what direction does the rate of change occur?

P 6. If $f(x, y, z) = zxe^y$, find the rate of change of f at the point $P(2, 0, 1)$ in the direction from P to $Q(1, 2, 3)$.

P 7. Find the directional derivative of the function

$$f(x, y, z) = x^2 y + x\sqrt{1 + z}$$

at $(1, 2, 3)$ in the direction of $\vec{v} = 2\vec{i} + \vec{j} - 2\vec{k}$.

P 8. If you are at the point $(2, 1, H(2, 1))$ on the surface $H(x, y) = x^2 - xy + 2y^2$, determine,

(a) the rate of change of H in the direction of the origin;

(b) a unit vector that points in the direction of steepest descent.

P 9. Assuming that the equation $x^2 + yz - y^2 + z^3 - 1 = 0$ determines a function $z = f(x, y)$, determine the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

P 10. Find an equation of the tangent plane to the surface $z = ye^{x/y}$ at $(1, 1)$.

P 11. Find the differential of

$$h(x, t) = e^{-3t} \sin(x + 5t)$$

P 12. An unevenly heated plate has temperature $T(x, y)$ in °C at the point (x, y). If $T(2, 1) = 135$, and $T_x(2, 1) = 16$, and $T_y(2, 1) = -15$, estimate the temperature at the point $(2, 04, 0, 97)$.

P 13. Find the best quadratic approximation for $f(x, y) = \ln(1 + x - 2y)$ for (x, y) near $(0, 0)$.

P 14. Find the quadratic Taylor polynomial valid near $(1, 0)$ for $f(x, y) = \sin(x - 1)\cos y$.

P 15. At what point on the surface $z = 1 + x^2 + y^2$ is the tangent plane parallel to the following planes?

$$z = 4 \text{ and } z = 2 + 6x - 4y.$$