

6 Relations

Name:

Date:

P 1. Let $A = \{1, 2, 3, 4, 5\}$

1. How many relations are there on A ?
2. How many reflexive relations are there on A ?

P 2. Decide if each of the following relations on the given set A is reflexive, symmetric, anti-symmetric and/or transitive. If not, explain.

1. $A = \{0, 1, 2\}$. $R_1 = \{(1, 2), (0, 2), (0, 1), (2, 1)\}$

2. $A = \mathbb{R}$. $R_2 : \{(x, y) \mid x^2 \geq 1\}$

3. $A = \Sigma^*$, where $\Sigma = \{a, b\}$. $R_3 = \{(w_1, w_2) \mid w_2^R = w_1^R\}$ Note: w^R is the reverse of w , e.g., $(aaba)^R = abaa$.

P 3. Let $A = \{a, b, c, d\}$. Find, if possible, a relation R on A containing (a, c) and (b, d) and which is symmetric and transitive but not reflexive. If not possible, **explain why not**.

P 4. Consider the following relations on A . Are they reflexive, symmetric, antisymmetric, or transitive? If they are, simply note this by putting the letter R, S, A , or T next to the relation. If not, *explain* why not.

1. $A = \{0, 1, 2, 3\}$. $R_1 = \{(0, 0), (0, 1), (0, 2), (0, 3), (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$.

2. $A = \mathbb{Z} - \{0\}$. $R_2 : \{(x, y) \mid \frac{x}{y} \in \mathbb{Z}\}$.

3. $A = \mathbb{Z}$. $R_3 : \{(x, y) \mid y = |x|\}$.

P 5. Let $A = \{0, 1, 2, 3\}$. Consider the relation $S = \{(0, 1), (1, 0), (1, 1), (2, 3), (3, 1)\}$ on A . Sketch the relation S .