Names of all group members: *Recitation section:*

Pledge and Signatures:

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1. (a) Use the Taylor series for $g(t) = e^t$ about $t = 0$ to obtain the Taylor series for $f(t) = te^t$ about $t = 0$.

(b) Using your answer to part (a), find a Taylor series expansion about $x = 0$ for

$$h(x) = \int_0^x te^t \, dt$$

(c) Use your answer to part (b) to show that

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4(2!)} + \frac{1}{5(3!)} + \frac{1}{6(4!)} + \cdots = 1.$$

2. In Enstein's theory of special relativity the mass of an object moving with velocity v is $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$ where m_0 is the mass of the object when at rest and c is the speed of light. Then the kinetic energy is defined as

$$K_{SR} = mc^2 - m_0c^2 = \frac{m_0c^2}{\sqrt{1 - v^2/c^2}} - m_0c^2 = m_0c^2 \left[\left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1 \right]. \tag{1}$$

In this exercise we will compare the special relativity notion of kinetic energy with the Newtonian one.

(a) Find the Taylor series of the binomial function $\dfrac{1}{\sqrt{1+x}} = (1+x)^{-1/2}$ around 0.

(b) Use part (a) and Equation (1) to write a series expansion for K_{SR} in terms of $\dfrac{v^2}{c^2}$.

(c) In Newtonian physics the kinetic energy of an object with velocity v and mass m_0 is $K_N = \frac{1}{2}m_0v^2$. Use the expansion found in part (b) for K_{SR} to argue that when v is very small compared to c one can assume $K_{SR} \approx K_N$. [Hint: You may neglect the terms involving $\left(\frac{v}{c}\right)^n$, $n \geq 4$, in the expansion for K_{SR}].