Ma 123 - Calculus 2A **Hw-2** **Due Feb. 6, 2015**

Name (Printed): *Recitation Section*:

Pledge and Sign:

Solutions are to be written up on a separate piece of paper, rather than directly on this cover sheet (unless explicitly allowed). Attach the cover sheet to your solution pages.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. The *sine integral* function is defined by the integral $Si(x) = \int_0^x \frac{\sin(t)}{t}\, dt$. Recall that $\frac{\sin(t)}{t}$ does not have elementary antiderivatives.

(a) Manipulate the Taylor series for $\sin(t)$ about $t = 0$ to find a series expansion for $\frac{\sin(t)}{t}$.

(b) Use the series obtained in part (a) to find a series expansion for $Si(x)$ about $x = 0$.

(c) Use the Taylor polynomial $P_7(x)$ of degree 7 for $Si(x)$ about $x = 0$ to estimate $Si(2)$.

2. Any object emits radiation when heated. A *blackbody* is a system that absorbs all the radiation that falls on it. For instance, a matte black surface on a large cavity with a small hole in its wall (like a blast furnace) is a blackbody and emits blackbody radiation. Even the radiation from sun is close to blackbody radiation.

Proposed in the 19'th century, the Rayleigh-Jeans Law expresses the energy density of a blackbody radiation of wavelength λ as

$$f(\lambda) = \frac{8\pi kT}{\lambda^4}$$

where λ is measured in meters, T is the temperature in kelvins (K), and k is Boltzmann's constant. The Rayleigh-Jeans Law agrees with experimental measurements for long wavelength but disagrees drastically for short wavelength. [The law predicts that $f(\lambda) \to \infty$ as $\lambda \to 0^+$ but experiments have shown that $f(\lambda) \to 0$.] This fact is known as *ultraviolet catastrophe.*

In 1900 Max Planck found a better model (known as Planck's law) for blackbody radiation:

$$f(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/(\lambda kT)} - 1}$$

where λ is measured in meters, T is the temperature in kelvins (K), and

$$\begin{aligned} h &= \text{Planck's constant} = 6.6262 \times 10^{-34}\ J \cdot s \\ c &= \text{speed of light} = 2.997925 \times 10^{8}\ m/s \\ k &= \text{Boltzmann's constant} = 1.3808 \times 10^{-23}\ J/K \end{aligned}$$

(a) Use L'Hospital's rule to show that

$$\lim_{\lambda \to 0^+} f(\lambda) = 0 \quad \text{and} \quad \lim_{\lambda \to \infty} f(\lambda) = 0$$

for Planck's law. [Hint: You may set $a = 8\pi hc$ and $b = hc/kT$ then $f(\lambda) = \dfrac{a\lambda^{-5}}{e^{b/\lambda} - 1}$. Also to evaluate the limits you may introduce a new variable μ and define $\mu = \lambda^{-1} = 1/\lambda$. Note that then $f(\lambda) = \dfrac{a\mu^5}{e^{b\mu} - 1}$, $\mu \to 0^+$ as $\lambda \to \infty$ and $\mu \to \infty$ whenever $\lambda \to 0^+$.]

(b) Use the Taylor series for e^x about 0 and the substitution $x = b/\lambda$ to show that for large wavelengths, Planck's law gives approximately the same values as the Rayleigh-Jeans Law.