Exam 1 Review Problems

Name: Date:

P 1. Given the infinite series

$$\sum_{n=1}^{\infty} a_n = \frac{2}{1} - \frac{3}{4} + \frac{4}{9} - \frac{5}{16} + \cdots$$

(a) Find a formula for the general term a_n.

(b) Find out if the series converges. Explain why.

P 2. Given the infinite series

$$\sum_{n=1}^{\infty} b_n = -3 + 1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots$$

(a) Find a formula for the general term b_n.

(b) Find out if the series converges. Explain why.

P 3. Determine whether the series are convergent or divergent. Explain.

(a) $\displaystyle\sum_{n=1}^{\infty} \tan\left(\frac{\pi}{4} + \frac{1}{n}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3 + \sin^2 n}{n^2 + 1}$

P 4. Find out if the series converges absolutely. Explain.

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\cos(2n)}{n^2}$$

P 5. A ball is dropped from a height of 10 feet. Each time it hits the ground, it bounces to 80% of its previous height. Find the total distance traveled by the ball.

P 6. Suppose

$$0 \leq b_n \leq \left(\frac{4}{3}\right)^n \leq a_n$$

and

$$0 \leq c_n \leq \left(\frac{3}{4}\right)^n \leq d_n$$

for all $n = 1, 2, 3, \cdots$ For each of the series

$$\sum_{n=1}^{\infty} a_n, \sum_{n=1}^{\infty} b_n, , \sum_{n=1}^{\infty} c_n, , \sum_{n=1}^{\infty} d_n$$

determine whether the series converges, diverges, or ther is not enough information to determine convergence or divergence. Justify your answers.

P 7. Find the Taylor series expansion for the following. Give your answer in compact form (i.e. find the general term).

(a) $f(x) = \cos\left(\frac{\pi x}{2}\right)$, centered at $a = 2$

(b) $f(x) = x\ln(1+x)$, centered at $a = 0$ (i.e. Maclaurin series).

P 8. Find the Taylor series for $f(x) = x^3$ about $x = 2$.

P 9. Let

$$f(x) = \int_0^x e^{t^2}\, dt$$

(a) Find the third-degree Taylor polynomial for $f(x)$ about $x = 0$.

(b) Estimate $f(0.5)$ using the Taylor polynomial found in part (a).

P 10. Let $f(x) = \sqrt{x}$

(a) Find the second-degree Taylor polynomial $P_2(x)$ for $f(x)$ about $x = 4$.

(b) Provide a bound for the error in the approximation $\sqrt{5} \approx P_2(5)$. Express your bound as a rational number. (You do not need to compute $P_2(5)$).

P 11. Consider the equation

$$\ln(1+x) = \frac{1}{2} - x$$

(a) Explain why the equation could have a solution for x close to 0.

(b) Estimate the solution of the equation using the second-degree Taylor polynomial for $\ln(1+x)$.

P 12. Find the Maclaurin series for

$$g(x) = \frac{e^x + e^{-x}}{2}$$

P 13. Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n}$$

P 14. Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x-a)^n}{nb^n},$$

(a and b are fixed constants with $b > 0$).

P 15. For what values of x does

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$$

converge?

P 16. Expand $f(x) = \dfrac{x^5}{x+3}$ into a power series centered at 0, and find its interval of convergence.

P 17. Find the sum of the series

$$\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n}}{6^{2n}(2n)!}$$

P 18. Determine the radius of convergence

$$\sum_{n=2}^{\infty} \frac{(-2)^n}{n \ln n}(x-1)^n$$

P 19. Find the limit

$$\lim_{x\to 0} \frac{e^x - \sin x - 1}{1 - \cos 2x}$$