4.3 Optimization and Modeling

Name: Date:

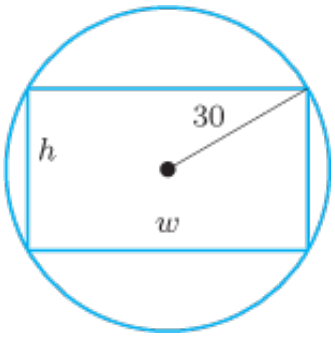
P 1. The sum of two nonnegative numbers is 100. What is the maximum value of the product of these two numbers?

P 3. The sum of three nonnegative numbers is 36, and one of the numbers is twice one of the other numbers. What is the maximum value of the product of these three numbers?

P 7. An open-topped rectangular box, with a square base x by x cm and height h cm. Find the dimensions giving the minimum surface area, given that the volume is 8 cm^3.

P 13. A right triangle has one vertex at the origin and one vertex on the curve $y = e^{-x/3}$ for $1 \leq x \leq 5$. One of the two perpendicular sides is along the x-axis; the other is parallel to the y-axis. Find the maximum and minimum areas for such a triangle.

P 17. A rectangular beam is cut from a cylindrical log of radius 30 cm. The strength of a beam of width w and height h is proportional to wh^2. Find the width and height of the beam of maximum strength.



P 29. Find the point(s) on the ellipse

$$\frac{x^2}{9} + y^2 = 1$$

(a) Closest to the point $(2, 0)$.

(b) Closest to the focus $(\sqrt{8}, 0)$. [Hint: Minimize the square of the distance - this avoids square roots.]

P 30. What are the dimensions of the closed cylindrical can that has surface area 280 square centimeters and contains the maximum volume?

P 39. The cost of fuel to propel a boat through the water (in dollars per hour) is proportional to the cube of the speed. A certain ferry boat uses \$100 worth of fuel per hour when cruising at 10 miles per hour. Apart from fuel, the cost of running this ferry (labor, maintenance, and so on) is \$675 per hour. At what speed should it travel so as to minimize the cost *per mile* traveled?